A NOVEL REDUCED-ORDER FAULT RECONSTRUCTION APPROACH FOR MISMATCHED SYSTEMS

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Abstract

In this paper, a reduced-order fault reconstruction approach is proposed for a mismatched non-linear system with actuator faults, disturbances and uncertainties. First, such a system is separated into multiple sub-systems through state transformation, and then, reduced-order transformation is conducted on the system to transform the mismatched system into matched sub-systems to obtain a reduced-order model that is efficient for locally separating the effects of mismatched faults and disturbances. Then, a sliding-mode observer is designed for the reduced-order model by reconstructing the faults using the equivalent output injection principle. Finally, a fourth-order non-linear aircraft model is conducted to verify the effectiveness of the proposed method.

Key Words

Sliding mode observer, reduced-order reconstruction, fault reconstruction

1. Introduction

In studies of fault diagnosis based on observer, the sliding-mode observer is widely used because of its good robustness [1], [2]. Studies on matching systems, in which faults and disturbances are only present in a fully observable state, have achieved several results. These studies are based on Edwards' approach [3], which divides the state into observable and unobservable parts using coordinate transformation, where the unobservable state does not contain faults and disturbances and can be asymptotically converged to a range. Therefore, in the observable section, a sliding-mode observer is designed and its faults are reconstructed using an equivalent output injection [4].

External disturbances are common in reality systems [5], [6]. Tan and Edwards minimised the effect of disturbances [7] on fault reconstruction using LMI [8]. When a system contains faults and disturbances, their decoupling has been the focus of many studies, Chan *et al.* reconstructed with two sliding mode observers [9], Chua *et al.* used LMI to design the gain of the observer so that the disturbances are bounded [10]. A consensus braking algorithm with distributed sliding-mode observers is proposed, the observer is capable of converging in a finite time [11]. When the dimensions of a system matrix were under specified conditions, He *et al.* simultaneously reconstructed the faults and disturbances, the original system is transformed into two subsystems by coordinate transformation, and the fault and disturbance are separated [12]. Among the studies on non-linear systems, many advancements have been achieved in Lipschitz non-linear systems [13]–[15]. Progress has also been obtained on sensor faults [16], notably by equating sensor faults to actuator faults using low-pass filters [17], [18]. He *et al.* reconstructed systems where actuator and sensor are fault simultaneously by coordinate transformation [19]. Generally, the model of fault reconstruction is developed from simple to complex to close the engineering background [20], and the limitations of fault reconstruction are becoming relaxed [21].

There are mismatches in reality systems, in which the unobservable state comprises a fault or disturbance. The investigation of such systems is crucial for practical applications in fault reconstruction. The mismatched faults or disturbances are described as mismatched terms, and the unobservable state will diverge under the influence of the mismatched terms, rendering the state untraceable and the residuals non-converging, failing fault reconstruction. Research has also been conducted on such systems. For graded faults, Tan and Edwards [22] employed a high-order sliding mode differentiator to generate a derivative of the output as a feedback input to the original

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system. Zhu and Yang [23] employed a cascaded sliding-mode observer using fault and observer signals as separate virtual system inputs and outputs and then periodically designed the observer for the virtual system until the conditions were met. Both approaches can yield certain results, but the structures are complicated. Zhirabok *et al.* designed reduced-order transformation for discrete systems [24], based on which actuator faults in linear systems are reconstructed [25]. The primary idea of reduced order is to transform a system model into a reduced-order model that does not contain a disturbance [26]. However, the fault is reconstructed using the Walcott–Zak observer's assumptions [27]. It has been demonstrated [28] that this assumption is a sufficient condition for matching systems. The reduced-order model essentially does not take into account the matching of reduced fault. In addition, the process of solving the parameters of the reduced-order model is slightly complicated. This paper contributes to the study of the reduced-order reconstruction of mismatched systems based on the traditional reduced-order model.

In this study, state transformation was performed first for the system model, and then, the sub-system that was not affected by the mismatched terms was adopted as the reduced-order model. Variables with a matching disturbance were used, and the reduced-order model was unaffected by a mismatched fault following order reduction. The reduced-order model contained reduced-order fault and disturbance, for which a sliding-mode observer can be developed to reconstruct the fault or disturbance. The innovations of this study are as follows: a new approach for reconstructing faults using matching sub-systems in reduced order is proposed. The matching condition only considers the relationship between the output matrix and the fault matrix. In this study, the state matrix was also considered to establish an approach for obtaining matching sub-systems in mismatched systems. Then, the observer was designed to build an approach for reconstructing faults in mismatched systems with reduced order. 2) The nature of the traditional reduced-order model was examined, and the design of the model was enhanced. In this study, the matching sub-systems were obtained using state transformation and were used as reduced-order systems to prevent the failure to satisfy the matching condition for the fault in the model. Meanwhile, matching disturbance was preserved as much as possible. The available order of the reduced-order model was increased to enhance reconstruction feasibility after reducing the order. Furthermore, the reducing order was simplified to make the selection of the transformation matrix easier.

This paper is structured as follows. Section 2 describes the system, designs the transformation matrix, obtains the matching sub-system as a reduced-order model, examines the traditional reduced-order model and states the paper's improvement. Section 3 establishes a sliding-mode observer for the new state equations, proves stability and reconstructs the faults and disturbances out by the equivalent output injection principle. Section 4 simulates the system to verify the model's validity, and Section 5 concludes this study.

Throughout this study, the notation $||*||$ denotes the Euclidean norm for vectors and (induced) spectral norm for matrices.

2. System Description

For mismatched systems containing a Lipschitz non-linear term, the actuator faults, disturbances, and uncertainties are termed as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + \Psi(x,u) + Bu(t) + Ef_a(t) + Dd(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the system state parameter, $u(t) \in R^r$ represents the input of the system, $y(t) \in R^p$ is the output signal of the system, and $n > p$. $f_a(t) \in R^q$ is an unknown bounded non-linear function that indicates system actuator faults, *i.e.*, there is a positive scalar function γ_1 which makes $||f_a(t)|| \leq \gamma_1$. $d(t) \in R^w$ denotes the unknown input disturbances and uncertainties of the system, and $||d(t)|| \leq \gamma_2$. $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{D}$, and $\boldsymbol{E}$ are known matrices, and $A \in R^{n\times n}$, $B \in R^{n\times r}$, $C \in R^{p\times n}$, $D \in R^{n\times w}$ and $E \in R^{n\times q}$.

The difficulty with analysing mismatched systems is that when the Edwards' approach is employed [3], the mismatched system (1) transforms into the following form:

$$\begin{aligned} \dot{x}_{\mathrm{I}}(t) &= A_{\mathrm{I}}x_{\mathrm{I}} + A_{\mathrm{II}}x_{\mathrm{II}} + \Psi_{\mathrm{I}}(x_{\mathrm{I}},u) + B_{\mathrm{I}}u + E_{\mathrm{I}}f_a + D_{\mathrm{I}}d \\ \dot{x}_{\mathrm{II}}(t) &= A_{\mathrm{III}}x_I + A_{\mathrm{IV}}x_{\mathrm{II}} + \Psi_{\mathrm{II}}(x_{\mathrm{II}},u) + B_{\mathrm{II}}u + E_{\mathrm{II}}f_a + D_{\mathrm{II}}d \\ \widetilde{y}(t) &= x_{\mathrm{II}}(t) \end{aligned} \tag{2}$$

The presence of mismatched terms prevents convergence x_I and thus affects fault reconstruction. In this study, the fault was reconstructed by solving the matching sub-system.

Assumption 1 [3]: The invariant zero of the matrix pair (A, C) is stable.

Considering the influence of mismatches on the system using the state matrix and non-linear term, the system is divided into the following parts. The transformation matrices are T and S.

$$z(t) = \mathrm{Tx}(t) = \left[z_1^T\ z_2^T\ z_3^T\ z_4^T\ z_5^T\right]^T, v(t) = \mathrm{Sy}(t) \tag{3}$$

$$\begin{aligned} \mathrm{TA}T^{-1} &= \overline{A}, \mathrm{SC}T^{-1} = \overline{C} = [0\ I_p], TB = \overline{B}, \\ TE &= \overline{E}, TD = \overline{D}, T\Psi = \overline{\Psi} \end{aligned} \tag{4}$$

The equations of the transformed system are as follows:

$$\begin{aligned} \dot{z}(t) &= \overline{A}z(t) + \overline{\Psi}(z,u) + \overline{B}u(t) + \overline{E}f_a(t) + \overline{D}d(t) \\ v(t) &= \overline{C}z(t) \end{aligned} \tag{5}$$

Where

$$\overline{A} = \begin{bmatrix} \overline{A}_{11} & \overline{A}_{12} & \overline{A}_{13} & \overline{A}_{14} & \overline{A}_{15} \\ 0 & \overline{A}_{22} & \overline{A}_{23} & \overline{A}_{24} & \overline{A}_{25} \\ \overline{A}_{31} & \overline{A}_{32} & \overline{A}_{33} & \overline{A}_{34} & \overline{A}_{35} \\ \overline{A}_{41} & \overline{A}_{42} & \overline{A}_{43} & \overline{A}_{44} & \overline{A}_{45} \\ 0 & \overline{A}_{52} & \overline{A}_{53} & \overline{A}_{54} & \overline{A}_{55} \end{bmatrix} \overline{\Psi} = \begin{bmatrix} \overline{\Psi}_1(z_1, z_2, z_3, z_4, z_5, u) \\ \overline{\Psi}_2(z_2, z_3, z_4, z_5, u) \\ \overline{\Psi}_3(z_1, z_2, z_3, z_4, z_5, u) \\ \overline{\Psi}_4(z_2, z_3, z_4, z_5, u) \\ \overline{\Psi}_5(z_2, z_3, z_4, z_5, u) \end{bmatrix}$$

$$\overline{B} = \begin{bmatrix} \overline{B}_1 \\ \overline{B}_2 \\ \overline{B}_3 \\ \overline{B}_4 \\ \overline{B}_5 \end{bmatrix} \overline{E} = \begin{bmatrix} \overline{E}_1 \\ 0 \\ \overline{E}_3 \\ \overline{E}_4 \\ \overline{E}_5 \end{bmatrix} \overline{D} = \begin{bmatrix} \overline{D}_1 \\ 0 \\ \overline{D}_3 \\ \overline{D}_4 \\ \overline{D}_5 \end{bmatrix} \quad (6)$$

n^i is defined as the number of dimensions of the state parameter, $i = 1, 2, \ldots, 5$. $z_i(t) \in R^{n_i}$, $\overline{\Psi}_i \in R^{n_i}$, $\overline{B}_i \in R^{n_i \times r}$, $\overline{E}_i \in R^{n_i \times q}$, $\overline{D}_i \in R^{n_i \times w}$ and $I_p \in R^{p \times p}$. z_1 is an unobservable state that is affected directly or indirectly by the mismatch term and state matrix. z_2 is an unobservable state and is unaffected by the mismatch term. In the observable state, z_3 and z_4 are affected by z_1. The difference is that the non-linear term $\overline{\Psi}_4$ in z_4 is unaffected by z_1, and neither is z_5 by z_1.

Assumption 2 [12]: $n_5 > q + w$.

Note: When Assumption 2 is met, the matching sub-system can reconstruct both the faults and disturbances.

On the basis of the transformation (4), (5) is further transformed by the following:

$$x_*(t) = \Gamma z(t), y_*(t) = \mathrm{Mv}(t) \quad (7)$$

Then, it is reduced to a matching sub-system:

$$\begin{cases} \begin{bmatrix} \dot{x}_{*1} \\ \dot{x}_{*2} \end{bmatrix} = \begin{bmatrix} A_{*1} \\ A_{*2} \end{bmatrix} x_{*1} + \begin{bmatrix} J_{*1} \\ J_{*2} \end{bmatrix} \overline{y} + \begin{bmatrix} \Psi_{*1}(x_{*1}, y_*, u) \\ \Psi_{*2}(x_{*1}, y_*, u) \end{bmatrix} \\ \qquad + \begin{bmatrix} B_{*1} \\ B_{*2} \end{bmatrix} u + \begin{bmatrix} 0 \\ E_{*2} \end{bmatrix} f_a + \begin{bmatrix} 0 \\ D_{*2} \end{bmatrix} d \\ y_* = x_{*2} \end{cases} \quad (8)$$

When F leads to $F\overline{A}_{41} = 0$, the rank of F is written as $rank(F) = \zeta$.

Let

$$\Gamma = \begin{bmatrix} 0 & I_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & F & 0 \\ 0 & 0 & 0 & 0 & I_5 \end{bmatrix}, M = \begin{bmatrix} 0 & F & 0 \\ 0 & 0 & I_5 \end{bmatrix} \quad (9)$$

where $I_2 \in R^{n_2 \times n_2}$ and $I_5 \in R^{n_5 \times n_5}$ are identity matrices. Then

$$\left[\frac{A_{*1}}{A_{*2}}\right] = \begin{bmatrix} \overline{A}_{22} \\ F\overline{A}_{42} \\ \overline{A}_{52} \end{bmatrix} \left[\frac{J_{*1}}{J_{*2}}\right] = \begin{bmatrix} \overline{A}_{23} \quad \overline{A}_{24}\overline{A}_{25} \\ F\overline{A}_{43} \quad F\overline{A}_{44} \quad F\overline{A}_{45} \\ \overline{A}_{53} \quad \overline{A}_{54} \quad \overline{A}_{55} \end{bmatrix}$$

$$\left[\frac{A_{*1}}{A_{*2}}\right] = \begin{bmatrix} \overline{B}_2 \\ F\overline{B}_4 \\ \overline{B}_5 \end{bmatrix} E_{*2} = \left[\frac{F\overline{E}_4}{E_5}\right] D_{*2} = \begin{bmatrix} F\overline{D}_4 \\ \overline{D}_5 \end{bmatrix} \quad (10)$$

In particular, when there is no $F\overline{A}_{41} = 0$ for any F, *i.e.*, $\overline{A}_{41}$ is a non-singular term, let

$$\Gamma = \begin{bmatrix} 0 \ I_2 0 \ 0 \ 0 \\ 0 \ 0 \ 0 \ 0 \ I_5 \end{bmatrix}, \ M = [0 \ 0 \ I_5], \quad (11)$$

$$\begin{bmatrix} A_{*1} \\ A_{*2} \end{bmatrix} = \begin{bmatrix} \overline{A}_{22} \\ \overline{A}_{52} \end{bmatrix}, \quad \begin{bmatrix} J_{*1} \\ J_{*2} \end{bmatrix} = \begin{bmatrix} \overline{A}_{23} & \overline{A}_{24} & \overline{A}_{25} \\ \overline{A}_{53} & \overline{A}_{54} & \overline{A}_{55} \end{bmatrix},$$

$$\begin{bmatrix} B_{*1} \\ B_{*2} \end{bmatrix} = \begin{bmatrix} \overline{B}_2 \\ \overline{B}_5 \end{bmatrix} E_{*2} = \overline{E}_5 D_{*2} = \overline{D}_5 \quad (12)$$

then:

$$A_* = \begin{bmatrix} A_{*1} \\ A_{*2} \end{bmatrix} J_* = \begin{bmatrix} J_{*1} \\ J_{*2} \end{bmatrix} B_* = \begin{bmatrix} B_{*1} \\ B_{*2} \end{bmatrix} E_* = \begin{bmatrix} 0 \\ E_{*2} \end{bmatrix} D_* = \begin{bmatrix} 0 \\ D_{*2} \end{bmatrix}$$
$$C_* = [0 \ I_c] \quad I_c \in R^{\theta \times \theta}, \theta = n_5 + \zeta \quad (13)$$

Equation (8) is taken as the reduced-order model

$$\begin{cases} \dot{x}_* = A_* x_{*1}(t) + J_* \overline{y} + \Psi_*(x_{*1}, y_*, u) + B_* u \\ \qquad + E_* f_a(t) + D_* d(t) \\ y_* = C_* x_*(t) \end{cases} \quad (14)$$

The reduced-order model is obtained by first transforming (1) and then reducing the order of (5). The reduced-order model satisfies the matching conditions. The reduced-order transformation can be expressed as the following:

$$\Gamma\overline{A} = A_*\Gamma + J_*\overline{C}, M\overline{C} = C_*\Gamma, \Gamma E = E_*, \Gamma D = D_* \quad (15)$$

In the process of reduced-order reconstruction, the original state parameters diverge under the influence of the mismatched term, the reduced-order state parameters are dynamically observed, and the deviation dynamic equations converge and become the basis of fault reconstruction.

3. Analysis of the Traditional Reduced-order Model

The primary idea of the traditional reduced-order model is to make the reduced-order model free of disturbances using reduced-order transformation, and Zhirabok transformed (1) into the form of (14) by performing the following state transformation [26]:

$$\Phi A = A_*\Phi + J_* C, R_* C = C_* \Phi \Phi E = E_*, \Phi D = D_* \equiv 0 \quad (16)$$

where Φ_i is the row i of Φ. Then, it is assumed that the fault is matched. The goal is to obtain a matching sub-system without disturbance through reduced-order transformation. In the design of the traditional reduced-order model, it is more difficult to determine all parameters simultaneously. For simplicity, the matrix below was given in [26]:

$$A_* \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}, \quad C_* = [1 \;\; 0 \;\; 0 \;\; \dots \;\; 0] \quad (17)$$

When $\Phi_i E \neq 0$ holds only to $i = 1$, (14) satisfies the matching condition. In this study, state transformation was performed first to obtain a matching sub-system, and then, the reduced-order model's parameters were selected accordingly, resulting in an equation that meets the matching condition:

$$C_* E_* = [0 \;\; I_c]\,, \begin{bmatrix} 0 \\ E_{*2} \end{bmatrix} = E_{*2} \quad (18)$$

The solving process was simpler and more intuitive.

As it is required $\Phi D = D_* \equiv 0$ in (16), in the reduced-order transformation, the observable state containing disturbance will be partially omitted according to the row rank of D_{*2}. When D_{*2} is a non-singular matrix, x_* will be fully rounded off. This reduces the degrees of freedom available for reduced-order reconstruction. In this study, the observable variables containing the matching disturbances were retained while the degree of freedom of the matching sub-system was increased.

In summary, the proposed approach of performing state transformation first and then reducing the order to obtain the parameters of the reduced-order model is more appropriate because it avoids the mismatched fault. The order reduction process is easier and more intuitive. It partially retains the state containing matching disturbance simultaneously, making reduced-order reconstruction more feasible.

4. Observer Design

A sliding-mode observer is designed for (8):

$$\begin{cases} \begin{bmatrix} \dot{\widehat{x}}_{*1} \\ \dot{\widehat{x}}_{*2} \end{bmatrix} = \begin{bmatrix} A_{*1} \\ A_{*2} \end{bmatrix} \widehat{x}_{*1} + \begin{bmatrix} J_{*1} \\ J_{*2} \end{bmatrix} \overline{y} + \begin{bmatrix} \widehat{\Psi}_{*1}(\widehat{x}_{*1}, \widehat{y}_*, u) \\ \widehat{\Psi}_{*2}(\widehat{x}_{*1}, \widehat{y}_*, u) \end{bmatrix} \\ \qquad + \begin{bmatrix} B_{*1} \\ B_{*2} \end{bmatrix} u + \begin{bmatrix} 0 \\ v(t) \end{bmatrix} - \begin{bmatrix} 0 \\ L \end{bmatrix} e_y \\ \widehat{y}_* = \widehat{x}_{*2} \end{cases} \quad (19)$$

where $e_y = y_* - \widehat{y}_*$ and $e = x_* - \widehat{x}_* = [e_1 \;\; e_2]^T$. As x_{*2} is fully observable, there must be an L, which makes A_{*0} = -L a stable matrix. $v(t)$ is the input signal of the sliding-mode observer:

$$v = \begin{cases} g \frac{Pe_y}{\|Pe_y\|} & \text{if } e_y \neq 0 \\ 0 & \text{otherwise} \end{cases} \quad (20)$$

where g is the scalar function to be designed. A_{*0} is a stabilising matrix. Therefore, for any symmetric matrix $Q > 0$, the Lyapunov equation

$$A_0^T P + P A_0 = -Q \quad (21)$$

has a unique solution P, which is the symmetric positive definite.

On the basis of (8) and (19), the deviation dynamic equation is written as the following:

$$\begin{bmatrix} \dot{e}_1(t) \\ \dot{e}_2(t) \end{bmatrix} = \begin{bmatrix} A_{*1} & 0 \\ A_{*2} & A_{*0} \end{bmatrix} \begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix} + \begin{bmatrix} \Psi_{*1} - \widehat{\Psi}_{*1} \\ \Psi_{*2} - \widehat{\Psi}_{*2} \end{bmatrix} + \begin{bmatrix} 0 \\ E_{*2} f_a + D_{*2} d - \;v(t) \end{bmatrix} \quad (22)$$

Assumption 3 [29]: Because $\Psi_{*1}(x_{*1}, y_*, u)$ is a Lipschitz non-linear function and y_* is observable, there exists a positive Lipschitz gain γ_3, γ_4 for the following:

$$\begin{aligned} \left\| \Psi_{*1}(x_{*1}, y_*, u) - \widehat{\Psi}_{*1}(\widehat{x}_{*1}, \widehat{y}_*, u) \right\| &\leq \gamma_3 \|e_1\| \\ \left\| \Psi_{*2}(x_{*1}, y_*, u) - \widehat{\Psi}_{*2}(\widehat{x}_{*1}, \widehat{y}_*, u) \right\| &\leq \gamma_4 \|e_1\| \end{aligned} \quad (23)$$

Theorem 1: e_1 is bounded and converges to a range after a certain amount of time t_0, i.e.,

$$\|e_1\| < \delta_1, \quad t > t_0 \quad (24)$$

where δ_1 is a positive scalar.

Proof: By Assumption 1 and referring to Yan 2008 [30], e_1 is bounded. □

Theorem 2: If the sliding mode input term g is chosen to be $g > (\|A_{*2}\| + \gamma_4)\delta_1 + \|E_{*2}\| \gamma_1 + \|D_{*2}\| \gamma_2 = g_1$, the deviation $\mathrm{e_y(t)}$ converges asymptotically to zero.

Proof: Given the Lyapunov function

$$V_1 = e_y^T \mathrm{P} e_y \quad (25)$$

take the derivative of (25) and substitute the deviation in (22) into it

$$\begin{aligned} \dot{V}_1 &= e_y^T \left(A_{*0}^T P + P A_{*0} \right) e_y + 2 e_y^T P \left(A_{*2} e_1 + \left(\Psi_{*2} - \widehat{\Psi}_{*2} \right) \right) \\ &\quad + 2 e_y^T P E_{*2} f_a + 2 e_y^T P D_{*2} d - 2 e_y^T \mathrm{Pv}(t) \\ \dot{V}_1 &= -e_y^T Q e_y + 2 e_y^T P \left(A_{*2} e_1 + \left(\Psi_{*2} - \widehat{\Psi}_{*2} \right) \right) \\ &\quad + 2 e_y^T P E_{*2} f_a + 2 e_y^T P D_{*2} d - 2 \mathrm{gP} e_y \\ \dot{V}_1 &= -e_y^T Q e_y + 2 P e_y \left(\left(A_{*_2} + \gamma_4 \right) \delta_1 + E_{*2} f_a + D_{*2} d - g \right) \end{aligned} \quad (26)$$

When $g > (\|A_{*2}\| + \gamma_4)\delta_1 + \|E_{*2}\|\gamma_1 + \|D_{*2}\|\gamma_2 = g_1$, there is

$$\dot{V}_1 \leq 0 \tag{27}$$

It is known from Theorem 2 that e_y is bounded, which means that there is a moment t_1 for the following:

$$\|e_y\| < \delta_2, \quad t > t_1 \tag{28}$$

where δ_2 is a positive scalar.

Theorem 3: If g is $g > (\|A_0\|\delta_2 + (\|A_{*2}\| + \gamma_4)\delta_1 + \|E_*\|\gamma_0 + \|D_{*2}\|\gamma_2)/\lambda_{\min}(P) = g_2$, then starting from any state point other than the slide mode surface $s = 0$, the system will reach the slide mode surface in a finite time.

Proof: Given the slide mode surface $s = Pe_y$ and build a Lyapunov function:

$$V_2 = \frac{1}{2}s^T s \tag{29}$$

Take the derivative of (29) and substitute the deviation in (22) into it

$$\begin{aligned}\dot{V}_2 &= s^T(P\dot{e}_y)\\ &= s^T P\Big(A_{*_2}e_1 + A_{*0}e_y + \Big(\Psi_{*2} - \widehat{\Psi}_{*2}\Big)\\ &\quad +E_{*2}f_a + D_{*2}d - v(t)\Big)\\ &= s^T P\Big(A_{*_2}e_1 + A_{*0}e_y + \Big(\Psi_{*2} - \widehat{\Psi}_{*2}\Big)\\ &\quad +E_{*2}f_a + D_{*2}d\Big) - s^T \mathrm{Pv}(t)\\ &= s^T P\Big(A_{*2}e_1 + A_{*0}e_y + \Big(\Psi_{*2} - \widehat{\Psi}_{*2}\Big)\\ &\quad +E_{*2}f_a + D_{*2}d\Big) - gs^T P\frac{Pe_y}{Pe_y}\end{aligned} \tag{30}$$

As $\lambda_{\min}(P)\|s\|^2 \leq s^T \mathrm{Ps}$

$$\begin{aligned}\dot{V}_2 \leq\ & s^T P\Big(A_{*_2}e_1 + A_{*0}e_y + \Big(\Psi_{*2} - \widehat{\Psi}_{*2}\Big) + E_{*2}f_a + D_{*2}d\Big)\\ & -g\lambda_{\min}(P)\parallel s\parallel \leq \parallel s\parallel \Big(\parallel P\parallel \Big(A_0\delta_2 + (A_{*_2} + \gamma_4)\delta_1\\ & +E_*\gamma_0 + D_{*2}\gamma_2\Big) - g\lambda_{\min}(P)\Big)\end{aligned} \tag{31}$$

When $g > (\|A_0\|\delta_2 + (\|A_{*2}\| + \gamma_4)\delta_1 + \|E_*\|\gamma_0 + \|D_{*2}\|\gamma_2)/\lambda_{\min}(P) = g_2$,

$$\dot{V}_2 \leq 0 \tag{32}$$

In summary, when slide mode input g is larger than g_1 and g_2, the convergence of the observation error is ensured and the system enters the sliding mode surface in a finite time. When the sliding motion is generated, there is $S = \dot{S} = 0$. According to the principle of equivalent output injection, the generalised fault will be reconstructed as the following:

$$0 = E_{*2}f_a + D_{*2}d - v_{\text{eq}} \tag{33}$$

$$v_{\text{eq}} = g\frac{Pe_y}{\|Pe_y\| + \delta} \tag{34}$$

where δ is a small positive scalar that suppresses the chattering of the sliding-mode motion. Let $\widetilde{E} = [E_{*2}\ D_{*2}]$, $f = \left[f_a^T\ d^T\right]^T$, then

$$f = -g(\widetilde{E}^T\widetilde{E})^{-1}\widetilde{E}^T\frac{Pe_y}{\|Pe_y\| + \delta} \tag{35}$$

f_a and d can be reconstructed from (35).

5. Simulation Example

To verify the effectiveness of the proposed approach in this study, the High Incidence Research Model aircraft model was adopted [30], which contains actuator faults and disturbances. According to (1), its parameters are

$$A = \begin{bmatrix} 0 & 0 & 0 & 1\\ -0.367 & -0.0318 & 0.0831 & -0.008\\ 0 & -0.0716 & -1.485 & 0.9848\\ 0 & -0.2797 & -5.672 & -1.0253\end{bmatrix}, B = \begin{bmatrix} 0 & 0\\ 0.012 & -0.0071\\ -0.3058 & -0.0223\\ -22.429 & 7.8777\end{bmatrix},$$

$$E = \begin{bmatrix}1 & 0\\ 0 & 1\\ 4 & 1\\ 1 & 0\end{bmatrix}, D = \begin{bmatrix}1\\0\\0\\0\end{bmatrix} C = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}, \Psi(x,u,t) = \begin{bmatrix}0 & \frac{\sin x_3}{(x_2+1)} & 0\end{bmatrix}^T$$

When this system is solved using the traditional reduced-order model method, the result is

$$\Phi = \begin{bmatrix}0 & 0 & 0.28 & 0.07\\ 0 & 0 & 0.8 & -0.2\end{bmatrix}, \quad E_* = \begin{bmatrix}1.19 & 0.28\\ 0 & 0.8\end{bmatrix},$$

$$C_* = \begin{bmatrix}1 & 0\end{bmatrix}$$

Obviously, there is $rank(C_*E_*) \neq rank(E_*)$, the matching condition for the fault is not satisfied. The fault cannot be reconstructed.

When using the approach proposed in this study, taking

$$\Gamma = \begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}, M = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

Then, the parameter matrix after transforming (14) is

$$A_* = 0, D_* = 0, J_* = \begin{bmatrix}-1.485 & 0.9848\\ -5.6725 & -1.0253\end{bmatrix},$$

$$B_* = \begin{bmatrix}-0.3058 & -0.0223\\ -22.4293 & 7.8777\end{bmatrix}, E_* = \begin{bmatrix}4 & 1\\ 1 & 0\end{bmatrix},$$

$$x_{1*} = x_3, x_{2*} = x_4$$

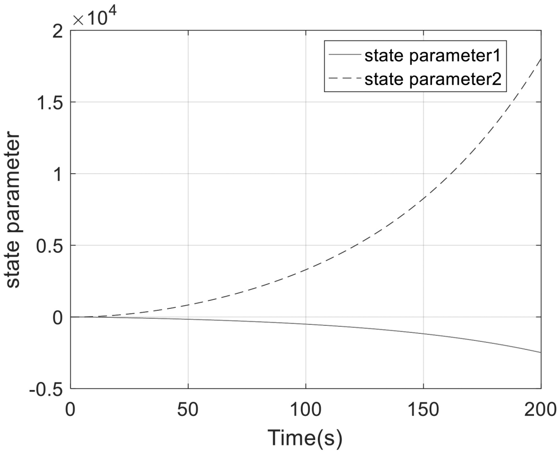

Figure 1. State parameters x_1 and x_2 of the original equation.

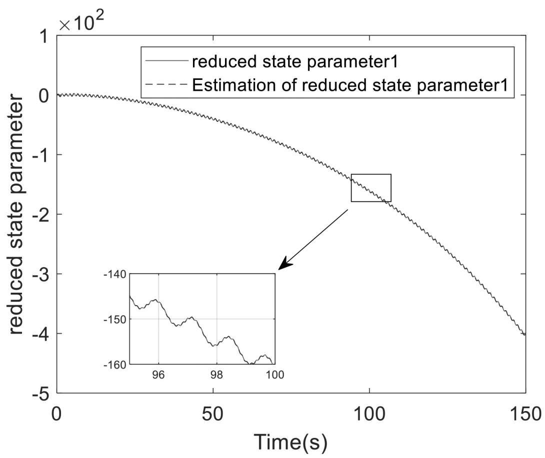

Figure 2. Reduced state parameter x_{*1}.

The observer parameters are set as follows:

$$L = \begin{bmatrix} -0.5 & 0 \\ 0 & -0.5 \end{bmatrix}, P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The two inputs are $u_1 = \sin(5t)$ and $u_2 = 4\sin(10t)$. As two actuator faults are involved, $f_{a1} = 2(40t) + 2\sin(5t)$ is used to simulate a gradual fault, and a step signal $f_{a2} = 3 * step(t-5)$ is adopted to simulate an abrupt fault. The disturbance d is the white noise of cycle 0.1 s and amplitude 1. δ is set to 0.07, and $g = 50$. The simulation results are as follows:

Figure 1 shows the actual values of the original equation's state parameters x_1 and x_2. Before order reduction, state parameters are not observable, so only x_1 and x_2 values are given. Figures 2 and 4 demonstrate the comparison between the actual and observed values of reduced state parameters x_{*1} and x_{*2}. Figures 3 and 5 show the observation errors for x_{*1} and x_{*2}, respectively. Under the influence of the mismatched terms, the state parameter x diverges, but the reduced state parameters x_{*1} and x_{*2} are observable

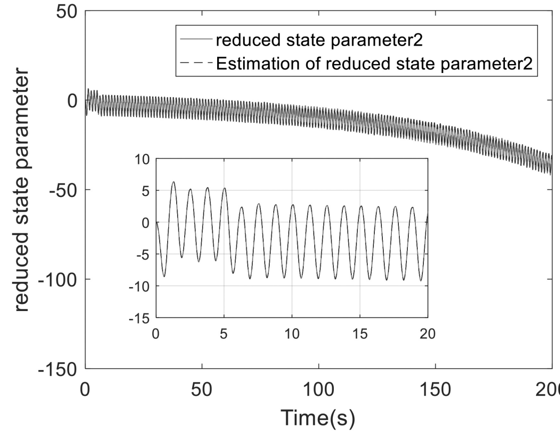

Figure 3. Reduced state parameter x_{*2}.

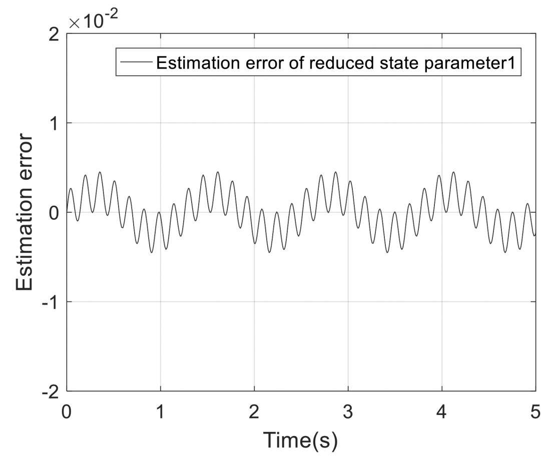

Figure 4. Observation error e_1 of reduced state x_{*1}.

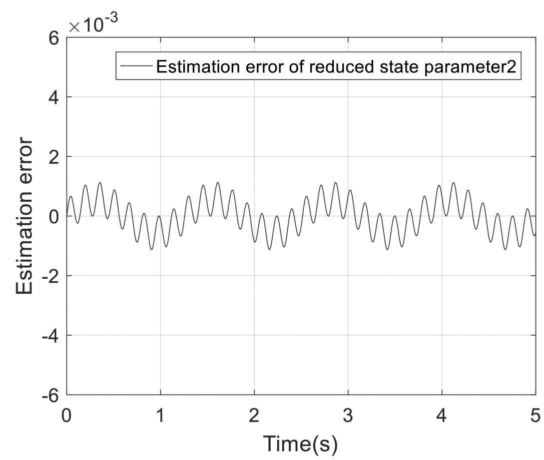

Figure 5. Observation error e_2 of reduced state x_{*2}.

with very small observation errors. The reduced state parameters x_{*1} and x_{*2} can be used to reconstruct the fault. The results of the fault reconstruction are as follows:

Where Figs. 6 and 8 show the actual versus observed values for actuator faults f_{a1} and f_{a2}. Figures 7 and 9

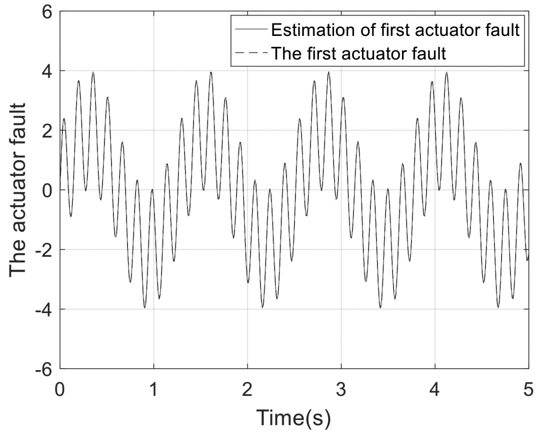

Figure 6. Actuator fault f_{a1}.

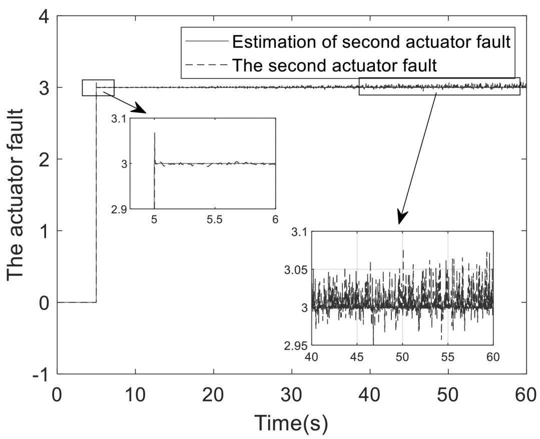

Figure 7. Actuator fault f_{a2}.

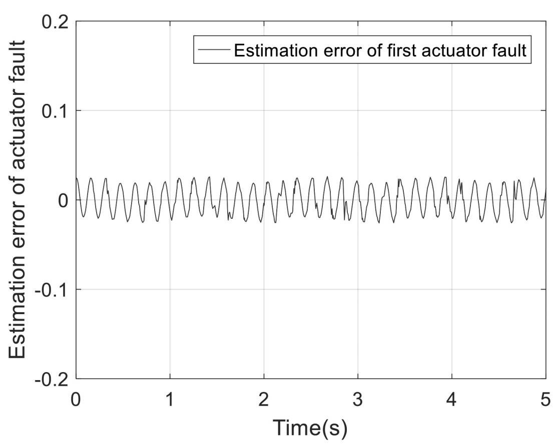

Figure 8. Observation error of actuator fault f_{a1}.

demonstrate the corresponding observation errors. It can be seen clearly from the above figures that the error between the fault and reconstructed value is very small. The approach proposed in this study can reconstruct abrupt and soft faults.

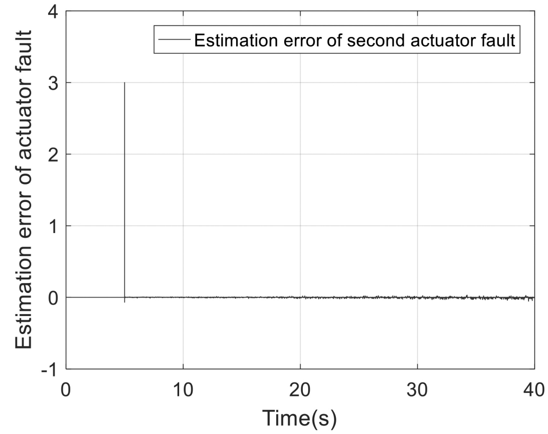

Figure 9. Observation error of actuator fault.

6. Conclusion

In this study, a new reduced-order fault reconstruction approach was established for mismatched non-linear systems. The approach considers the effect of the mismatch terms on the system through the state matrix and carries out state transformation on the system model to obtain a matching sub-system and treat it as a reduced-order model, for which a sliding-mode observer was designed to reconstruct the fault or disturbance contained within it. Then, the validity of the results was later demonstrated using simulation experiments. Owing to the reduced-order reconstruction's design characteristics, the terms that can be reconstructed are related to the faults and disturbances contained in the reduced-order model. If the orders of these faults and disturbances exceed the orders of the reduced parameters, it would be impossible to decouple the faults and disturbances. The reduced-order model's inadequacies will be further investigated and addressed.

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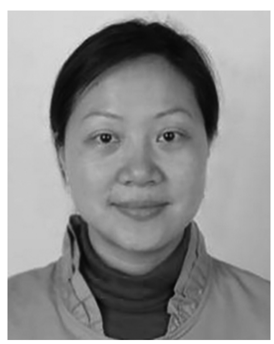

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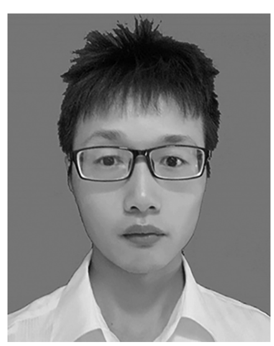

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