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ENHANCED EXTENDED STATE OBSERVER BASED OUTPUT-FEEDBACK TRACKING CONTROL OF WHEELED MOBILE ROBOT WITH DISTURBANCE

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Abstract

Considering the widespread disturbance in the wheeled mobile robot system, an enhanced extended state observer (EESO) is deployed, thus, the unmeasured states and disturbances can be estimated. Based on the estimated states and disturbance, an output-feedback controller is constructed. The estimated states and measured position information are used for the feedback part, and the estimated disturbances are used for the feedforward part. Lyapunov functions are built to prove that the observer error dynamics and the control error system satisfy input-to-state stable (ISS). All error signals are uniformly and ultimately bounded (UUB). Simulation results reveal the superiority of the designed method.

Key Words

Wheeled mobile robot, tracking control, enhanced extended state observer, external disturbances, output feedback

1. Introduction

Wheeled mobile robot has rapidly developed for decades, and it has been proven effective in many fields [1], such as the national defense industry, the transportation industry, *etc*. For a WMR system, trajectory tracking control is one of the fundamental functions. After path planning module generates the reference trajectory, the trajectory tracking module will calculate the wheels' moment to drive the WMR system. In the beginning, only the kinematics model is considered when designing the tracking controller. However, the WMR system relies on the dynamics model to produce the ideal velocities [2]–[4]. Thus, the dynamics model of WMR is not neglected. Due to the widespread disturbance, it is necessary to deal with the disturbance in the WMR dynamics model.

To conquer this problem, many control methods are proposed for the WMR system with dynamics disturbance. In [5], a model-free adaptive controller is constructed based on the state error. An event-observer is designed to estimate the unknown states, actuator faults, and disturbances for the WMR system [6]. In [7], the state error is estimated by an intermediate estimator, and a virtual system is built to generate the reference trajectory. An adaptive neural network is deployed to approximate the unknown WMR system parameter matrix [8]. In [9], the disturbance observer (DO) is applied for disturbances and uncertainties. Active disturbance rejection control (ADRC) can compensate for internal uncertainties, and only a few plant details are required. In fact, it only needs the system order of the plant. Dependent on that reason, ADRC, nowadays, is widely applied in lots of practical systems. Same to DO [10], the extended state observer (ESO), as the core part of ADRC, is a powerful method to estimate the lumped disturbance without any specific disturbance information [11]–[13]. For a perturbed system, the ESO-based control law has drawn much more attention nowadays[14], [15]. The reduced-order ESO is developed to estimate the unknown dynamic model of a WMR system [16]. In [17], an adaptive law is developed to resist the parameter uncertainties, and an ESO is developed to resist the external disturbance. An ESO based slide mode control method is deployed to estimate the unknown uncertainties [18]. In [19], a time-varying fixed-time ESO is designed for the leader WMR. In [20], the WMR model is transformed to a linear one by feedback linearisation, then, a finite-time ESO is developed to estimate the lumped disturbance. To estimate the unknown states and disturbance, neural networks or fuzzy-logic systems are combined with observer-based control [21]. However, these methods will consume more resources.

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In [22], the authors mentioned that disturbance denotes the difference between the system's stable state and its current state. As for a trajectory tracking system, its stable state means that the tracking error is 0. Any force that deviates the tracking error from its equilibrium point should be seen as a disturbance. If the trajectory tracking system is powered by an error feedback controller, the control input will tend to be 0 when the error is 0. Considering the randomness of the internal or external disturbance, it can be supposed that there is a part of the disturbance that acts as an input to stabilise the tracking system. This part of the disturbance can be seen as a favorable disturbance. And the other part of disturbance can be seen as the negative disturbance, which enlightens us that only the negative disturbance should be eliminated. Thus, how to estimate and compensate for the negative disturbance is another concern in this manuscript.

This paper takes the kinematics model and dynamics model into account and focuses on the disturbance that mainly exists in the dynamics model. The original system and the reference system are proposed to describe the current state and the stable state of the WMR trajectory tracking system. By constructing the ESOs for both systems, the unmeasured states, the lumped disturbance, and the favorable disturbance can be estimated. An enhanced ESO (EESO) is constructed to estimate the negative disturbance and the unmeasured state errors. With these estimations, an output-feedback controller is developed. The estimated state errors and the negative disturbance make up the feedback part and feedforward part. Both the purposes of trajectory tracking and disturbance compensation are fulfilled. The main contributions are summarised as follows. In contrast to the conventional ESO based control methods [11]–[15], an EESO is built based on both the reference system output and the actual system output to estimate the tracking error and negative disturbance. In addition, the designed control strategy will address the tracking problem for a more general system better than the classic ESOBC technique [23].

2. Problem Formulation

2.1 Kinematic and Dynamic Model of WMR

Figure 1 illustrates the model of the WMR. $[x,\ y]^T$ and φ are the body positionin the earth coordinate and the orientation between the X-axis and the WMR's forward direction. ν and ω denote the linear velocity and angular velocity of the WMR. r is the radius of the wheel, and $2R$ is the tread of the WMR.

Denote $[x\ y\ \varphi]^T$ as q, $\eta = [\nu\ \omega]^T$, the kinematic model of WMR is given as follows.

$$\dot{q} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} \cos\varphi & 0 \\ \sin\varphi & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \nu \\ \omega \end{bmatrix} = J(q)\eta \tag{1}$$

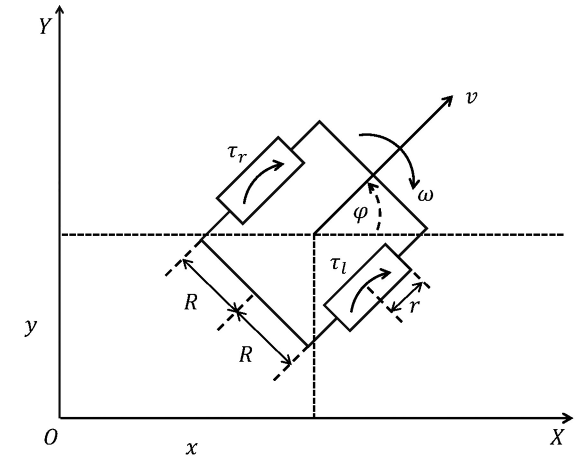

Figure 1. The model of the WMR.

Considering the wheels purely roll without slipping and skidding:

$$\begin{bmatrix} -\sin\varphi & \cos\varphi & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\varphi} \end{bmatrix} = 0 \tag{2}$$

Denote $[-\sin\varphi\ \cos\varphi\ 0]$ as $S(q)$, the dynamic model of WMR is given as follows.

$$M(q)\ddot{q} = B(q)\tau - S^T(q)\lambda + \tau_d \tag{3}$$

Where, $M(q) = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & j \end{bmatrix}$ is the inertia matrix, $B(q) = \begin{bmatrix} \cos\varphi/r & \cos\varphi/r \\ \sin\varphi/r & \sin\varphi/r \\ R/r & -R/r \end{bmatrix}$, and m is the mass, j is the moment of inertia of the body of WMR. $\tau_d = [\tau_1\ \tau_2\ \tau_3]^T$ denotes the external disturbance. $\tau = [\tau_r,\ \tau_l]^T$ is the input torque.

As $J^T(q)S(q) = 0$, multiply $J^T(q)$ to both side of (3), according to (1), the dynamic model is concluded as:

$$\dot{\eta} = \begin{bmatrix} \frac{1}{\mathrm{mr}} & \frac{1}{\mathrm{mr}} \\ \frac{R}{\mathrm{jr}} & \frac{-R}{\mathrm{jr}} \end{bmatrix} \tau + \begin{bmatrix} \tau_{\mathrm{dr}} \\ \tau_{\mathrm{dl}} \end{bmatrix} \tag{4}$$

where, $\eta = \begin{bmatrix} \nu & \omega \end{bmatrix}^T$, $\begin{bmatrix} \tau_{\mathrm{dr}} & \tau_{\mathrm{dl}} \end{bmatrix}^T = \begin{bmatrix} \frac{\tau_1\cos\varphi + \tau_2\sin\varphi}{m} & \frac{\tau_3}{j} \end{bmatrix}^T$.

Choose a heading position $(x_l,\ y_l)$ which is expressed by

$$\begin{bmatrix} x_l \\ y_l \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + l \begin{bmatrix} \cos\varphi \\ \sin\varphi \end{bmatrix} \tag{5}$$

where l is the distance between the body center position and the heading position. Differentiating (5) twice and substituting(1)(4) yields:

$$\begin{bmatrix}\ddot{x}_l \\ \ddot{y}_l\end{bmatrix} = \begin{bmatrix}\cos\varphi & -l\sin\varphi \\ \sin\varphi & l\cos\varphi\end{bmatrix}\dot{\eta} + \begin{bmatrix}-\nu\omega\sin\varphi - l\cos\varphi\omega^2 \\ -\nu\omega\cos\varphi - l\sin\varphi\omega^2\end{bmatrix}$$
$$= \frac{1}{r}\begin{bmatrix}\frac{\cos\varphi}{m} - \frac{\mathrm{lR}\sin\varphi}{j} & \frac{\cos\varphi}{m} + \frac{\mathrm{lR}\sin\varphi}{j} \\ \frac{\sin\varphi}{m} + \frac{\mathrm{lR}\cos\varphi}{j} & \frac{\sin\varphi}{m} - \frac{\mathrm{lR}\cos\varphi}{j}\end{bmatrix} \tag{6}$$
$$+ \begin{bmatrix}-\nu\omega\sin\varphi - l\cos\varphi\omega^2 + \frac{1}{r}\left(\tau_{\mathrm{dr}}\cos\varphi - l\tau_{\mathrm{dl}}\sin\varphi\right) \\ -\nu\omega\cos\varphi - l\sin\varphi\omega^2 + \frac{1}{r}\left(\tau_{\mathrm{dr}}\sin\varphi - l\tau_{\mathrm{dl}}\cos\varphi\right)\end{bmatrix} n$$

Let:

$$\begin{bmatrix}d_{\tau\mathrm{r}} \\ d_{\tau\mathrm{l}}\end{bmatrix} = \begin{bmatrix}-\nu\omega\sin\varphi - l\cos\varphi\omega^2 + \frac{1}{r}\left(\tau_{\mathrm{dr}}\cos\varphi - l\tau_{\mathrm{dl}}\sin\varphi\right) \\ -\nu\omega\cos\varphi - l\sin\varphi\omega^2 + \frac{1}{r}\left(\tau_{\mathrm{dr}}\sin\varphi - l\tau_{\mathrm{dl}}\cos\varphi\right)\end{bmatrix} \tag{7}$$

and $\Pi = \frac{1}{r}\begin{bmatrix}\frac{\cos\varphi}{m} - \frac{\mathrm{lR}\sin\varphi}{j} & \frac{\cos\varphi}{m} + \frac{\mathrm{lR}\sin\varphi}{j} \\ \frac{\sin\varphi}{m} + \frac{\mathrm{lR}\cos\varphi}{j} & \frac{\sin\varphi}{m} - \frac{\mathrm{lR}\cos\varphi}{j}\end{bmatrix}$, thus,

$$\begin{bmatrix}\ddot{x}_l \\ \ddot{y}_l\end{bmatrix} = \Pi\tau + \begin{bmatrix}d_{\tau\mathrm{r}} \\ d_{\tau\mathrm{l}}\end{bmatrix} \tag{8}$$

To avoid the torques coupling in (8), a new control inputs vector is assigned as:

$$U = \Pi\tau \tag{9}$$

where, $U = [u_x \; u_y]^T$.

Thus, the state space model of WMR can be expressed as:

$$\begin{cases}\dot{q}_{l1} = q_{l2} \\ \dot{q}_{l2} = U + D\end{cases} \tag{10}$$

where, $q_{l1} = [x_l, y_l]^T$, $q_{l2} = [\dot{x}_l, \dot{y}_l]^T$, and $D = [d_{\tau r}, d_{\tau l}]^T$.

Our purpose is to design a disturbance rejection tracking control protocol to achieve prescribed time tracking. Next, some assumptions are given as follows.

Assumption 1. *The system* (10) *is controllable.*

Assumption 2. *Denote the derivative of the disturbance D as H, where $H = [h_{\tau r}, h_{\tau l}]^T$, and $h_{\tau r}$, $h_{\tau l}$ satisfy the bounded condition, which means $\{|h_{tr}|, |h_{tl}| \le \bar{h}\}$, where $\bar{h}$ is a positive constant.*

Remark 1. *For state stabilising and signal tracking, the system* (10) *must satisfy the condition of controllability. Most common disturbances can be seen as a different common signal combination. Thus, the derivative of the external disturbance is continuous except for the step signal. The step signal can't be achieved in a practical system, which can be taken over by a ramp signal with an extremely large slope. So basically, the derivation of the external disturbance is continuous and bounded in the practical system.*

Lemma 1. *In [24], consider:*

$$\dot{x} = f(t, x, u) \tag{11}$$

where, $f(t, x, u)$: $\mathbb{R}\times\mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is continuously differentiable.

Suppose $\forall x \in \mathbb{R}^n, \forall u \in \mathbb{R}^m$, there exists a continuous differential function V: $\mathbb{R}^n \to \mathbb{R}^+$, which satisfies:

$$\zeta_1(|x|) \le V(t, x) \le \zeta_2(|x|) \tag{12}$$
$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t, x, u) \le -W(|x|), \forall |x| \geqslant \gamma(|x|) > 0 \tag{13}$$

where ζ_1 and ζ_2 are class $\mathcal{K}_\infty$ functions, γ is a $\mathcal{K}$ function, and $W \in \mathbb{R}^n$ is a positive continuous differential function. Then, the system (11) is input-to-state stable (ISS) with $\psi = \zeta_1^{-1} \circ \zeta_2 \circ \gamma$.

3. Output-Feedback Trajectory Tracking Control Law Design

3.1 Enhanced Extended State Observer

Considering the position of WMR is the only available information. The unavailable information and disturbance can be estimated by the ESO. In the proceeding section, two system models are developed based on the current and target system states. Then, an EESO is designed according to these two system models. The negative disturbance, which destabilises the controlled tracking system, can be estimated.

3.1.1 The Actual System

Define an actual system as:

$$\begin{cases}\dot{q}_{l1a} = q_{l2a} \\ \dot{q}_{l2a} = U + D_a\end{cases} \tag{14}$$

where, $q_{l1a} = q_{l1}$, $q_{l2a} = q_{l2}$, and $D_a = D$.

Define D_a as the system extended state q_{l3a}, it yields:

$$\begin{cases}\dot{q}_{l1a} = q_{l2a} \\ \dot{q}_{l2a} = U + q_{l3a} \\ \dot{q}_{l3a} = H_a\end{cases} \tag{15}$$

where, H_a is the derivative of D_a.

Assumption 3. *The extended system* (15) *is observable.*

An ESO for (15) is proposed to estimate the disturbance D_a as follows:

$$\begin{cases}\dot{\widehat{q}}_{l1a} = \widehat{q}_{l2a} - \varepsilon_1\left(\widehat{q}_{l1a} - q_{l1a}\right) \\ \dot{\widehat{q}}_{l2a} = U + \widehat{q}_{l3a} - \varepsilon_2\left(\widehat{q}_{l1a} - q_{l1a}\right). \\ \dot{\widehat{q}}_{l3a} = -\varepsilon_3\left(\widehat{q}_{l1a} - q_{l1a}\right)\end{cases} \tag{16}$$

where, ε_1, ε_2, and ε_3 are the observer gain matrix.

3.1.2 The Reference System

For the WMR trajectory tracking system, the tracking error is usually used for the error feedback controller. As long as the error tends to 0, which means the system has tracked the given trace, the control input tends to 0 as well. Considering a perturbed system, some parts of the disturbance act on the system to track the trajectory when the control input tends to 0. These parts of disturbance are favorable disturbances. In other words, the difference between the total disturbance and favorable disturbance is the negative disturbance, which needs to be eliminated. Therefore, the reference system model is given as:

$$\begin{cases} \dot{q}_{l1r} = q_{l2r} \\ \dot{q}_{l2r} = D_r \end{cases} \tag{17}$$

where, q_{l1r} is actually the reference trajectory, D_r is favorable disturbance.

Define D_r as the system extended state q_{l3r}, it yields:

$$\begin{cases} \dot{q}_{l1r} = q_{l2r} \\ \dot{q}_{l2r} = q_{l3r} \\ \dot{q}_{l3r} = H_r \end{cases} \tag{18}$$

where, H_r is the derivative of D_r.

Assumption 4. *The extended system* (18) *is observable.*

An ESO for (18) is proposed to estimate the disturbance D_a as follows:

$$\begin{cases} \dot{\widehat{q}}_{l1r} = \widehat{q}_{l2r} - \varepsilon_1 \left(\widehat{q}_{l1r} - q_{l1r}\right) \\ \dot{\widehat{q}}_{l2r} = \widehat{q}_{l3r} - \varepsilon_2 \left(\widehat{q}_{l1r} - q_{l1r}\right) \\ \dot{\widehat{q}}_{l3r} = -\varepsilon_3 \left(\widehat{q}_{l1r} - q_{l1r}\right) \end{cases} \tag{19}$$

where, ε_1, ε_2, and ε_3 are the observer gain matrix.

One of our main purposes is to distinguish the unfavorable disturbance that hinders the controlled system output from the given trajectory. In [22], Gao proposed that the disturbance can be deemed to be the difference between what a controlled system actually is and what it should be. Thus, an actual system (14) is defined to describe what a controlled system actually is, and a reference system (17) is defined to describe the "should-be" system. The two systems defined above represent different system states of the original controlled system (10). In fact, the reference system model denotes the "expected" steady state of the controlled system (10). Meanwhile, the reference model should satisfy the following conditions.

1) The reference system and the actual system have the same system matrices and structure.
2) A favorable disturbance replaces the control input signal to stabilise the reference system.
3) The reference system output is equivalent to the given reference signal.

3.1.3 Enhanced Extended State Observer

Based on (16)and (19), it is natural to substrate the favorable disturbance from the total disturbance to get the negative disturbance. Thus, an EESO is obtained:

$$\begin{cases} \dot{\widehat{\overline{q}}}_{l1} = \widehat{\overline{q}}_{l2} - \varepsilon_1 \left(\widehat{\overline{q}}_{l1} - q_{l1a} + q_{l1r}\right) \\ \dot{\widehat{\overline{q}}}_{l2} = U + \widehat{\overline{q}}_{l3} - \varepsilon_2 \left(\widehat{\overline{q}}_{l1} - q_{l1a} + q_{l1r}\right) \\ \dot{\widehat{\overline{q}}}_{l3} = -\varepsilon_3 \left(\widehat{\overline{q}}_{l1} - q_{l1a} + q_{l1r}\right) \end{cases} \tag{20}$$

where, $\overline{q}_{l1} = q_{l1a} - q_{l1r}$, $\widehat{\overline{q}}_{l1}$, is the estimation of $\overline{q}_{l1}$, $\overline{q}_{l2} = q_{l2a} - q_{l2r}$, and $\overline{q}_{l3} = q_{l3a} - q_{l3r}$.

Define the estimation error as:

$$e_{\text{oa}} = \begin{bmatrix} \widehat{q}_{l1a} - q_{l1a} \\ \widehat{q}_{l2a} - q_{l2a} \\ \widehat{q}_{l3a} - q_{l3a} \end{bmatrix} \quad e_{\text{or}} = \begin{bmatrix} \widehat{q}_{l1r} - q_{l1r} \\ \widehat{q}_{l2r} - q_{l2r} \\ \widehat{q}_{l3r} - q_{l3r} \end{bmatrix} \tag{21}$$

Thus,

$$\begin{aligned} e_o &= \begin{bmatrix} \widehat{\overline{q}}_{l1} - \overline{q}_{l1} \\ \widehat{\overline{q}}_{l2} - \overline{q}_{l2} \\ \widehat{\overline{q}}_{l3} - \overline{q}_{l3} \end{bmatrix} = \begin{bmatrix} \widehat{q}_{l1a} - q_{l1a} \\ \widehat{q}_{l2a} - q_{l2a} \\ \widehat{q}_{l3a} - q_{l3a} \end{bmatrix} - \begin{bmatrix} \widehat{q}_{l1r} - q_{l1r} \\ \widehat{q}_{l2r} - q_{l2r} \\ \widehat{q}_{l3r} - q_{l3r} \end{bmatrix} \\ &= e_{\text{oa}} - e_{\text{or}} \end{aligned} \tag{22}$$

From (15), (16), (18), (19), (21), and (22), the observer estimation error dynamic can be obtained:

$$\dot{e}_o = \dot{e}_{\text{oa}} - \dot{e}_{\text{or}} = A_o e_o + E\left(H_a - H_r\right) \tag{23}$$

where $A_o = \begin{bmatrix} -\varepsilon_1 & I & 0 \\ -\varepsilon_2 & 0 & I \\ -\varepsilon_3 & 0 & 0 \end{bmatrix} \in \mathbb{R}^{6\times 6}$, $E = \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix} \in \mathbb{R}^{6\times 2}$, H_a and H_r are the derivatives of D_a and D_r, separately.

For (23), if the observer gain matrix ε_1, ε_2, and ε_3 are chosen properly such that A_o is Hurwitz, then, there exist matrix $M = M^T > 0$ and any given matrix $P^T = P > 0$ satisfying

$$A_o^T M + M A_o = -P \tag{24}$$

Lemma 2. *If the Assumptions 1–4 are satisfied and* (24) *stands, then, the error dynamic system* (23) *with input being* $H_a - H_r$*, is ISS.*

Proof. Define a Lyapunov function:

$$V_0 = \frac{1}{2} e_o^T M e_o \tag{25}$$

which is bounded by:

$$\frac{1}{2}\lambda_{\min}(M)\left|e_o\right|^2 \le V_0 \le \frac{1}{2}\lambda_{\max}(M)\left|e_o\right|^2 \tag{26}$$

And its derivative is obtained:

$$\dot{V}_0 = -\frac{1}{2} e_o^T P e_o + e_o^T \text{ME}\left(H_a - H_r\right) \tag{27}$$

Based on Assumption 2, H_a, H_r are bounded. Thus,

$$\begin{aligned}\dot{V}_0 &\leq -\frac{1}{2}e_o^T P e_o + 2\overline{h}e_o^T \mathrm{ME} \\ &\leq -\frac{1}{2}\lambda_{\min}(P)\left|e_o\right|^2 + 2\overline{h}\left|e_o\right|\left|M\right|n \end{aligned} \tag{28}$$

As:

$$|e_o| \geqslant \frac{4\overline{h}|M|}{\lambda_{\min}\ (P)a} \tag{29}$$

$$\dot{V}_0 \leq -\frac{1}{2}(1-a)\lambda_{\min}\ (M)\left|e_o\right|^2 n \tag{30}$$

where $0 < a < 1$. Therefore, from Lemma 1, the estimation error dynamic system (23) with regard to the input $H_a - H_r$ is ISS, and:

$$|e_o| \leq \sqrt{\frac{\lambda_{\max}(M)}{\lambda_{\min}(M)}} \times \max\left(\left|e_o\left(t_0\right)\right| e^{-\gamma_1(t-t_0)}, \frac{4\overline{h}|M|}{\lambda_{\min}(P)a}\right), \forall t \geqslant t_0. \tag{31}$$

where $\gamma_1 = \frac{\lambda_{\min}(P)}{\lambda_{\max}(M)}(1-a)$. □

3.2 Output-Feedback Controller Design

The output-feedback controller is designed:

$$U = k_1\left(q_{l1a} - q_{l1r}\right) + k_2\left(\overline{q}_{l2}\right) - \overline{q}_{l3} \tag{32}$$

where $k_1 \in \mathbb{R}^{2\times 2}, k_1 \in \mathbb{R}^{2\times 2}$ are the control gain matrix.

From (9), the controlled torque is obtained as:

$$\tau = \Pi^{-1}U = \Pi^{-1}\left(k_1\left(q_{l1a} - q_{l1r}\right) + k_2\left(\overline{q}_{l2}\right) - \overline{q}_{l3}\right) \tag{33}$$

Define the control error as:

$$e_c = \begin{bmatrix} q_{l1a} - q_{l1r} \\ q_{l2a} - q_{l2r} \\ q_{l3a} - q_{l3r} \end{bmatrix} \tag{34}$$

From (14), (17), (20), and(32), it yields:

$$\dot{e}_c = A_c e_c + B_u e_o \tag{35}$$

where, $A_c = \begin{bmatrix} 0 & I \\ k_1 & k_2 \end{bmatrix} \in \mathbb{R}^{4\times 4},\ B_u = \begin{bmatrix} 0 & 0 & 0 \\ 0 & k_2 & -I \end{bmatrix} \in \mathbb{R}^{4\times 6}$.

Lemma 3. *If k_1, k_2 are chosen properly such that A_c is Hurwitz, then, the control error dynamic system* (35) *with the state being e_c and input being e_o is ISS.*

Proof. As A_o is Hurwitz, there exists a matrix $N^T = N > 0$ and any given matrix $Q^T = Q > 0$ satisfying:

$$A_c^T N + N A_c = -Q \tag{36}$$

Construct a Lyapunov function as:

$$V_1 = \frac{1}{2}e_c^T N e_c \tag{37}$$

Its derivative is obtained as:

$$\begin{aligned}\dot{V}_1 &= -\frac{1}{2}e_c^T Q e_c + e_c^T N B_u e_o \\ &\leq -\frac{1}{2}\lambda_{\min}(Q)\left|e_c\right|^2 + \left|e_c\right|\left\|N B_u\right\| e_o n \end{aligned} \tag{38}$$

As:

$$|e_c| \geqslant \frac{|NB_u|\,|e_o|}{\lambda_{\min}(N)b} \tag{39}$$

where $0 < b < 1$. It makes:

$$\dot{V}_1 \leq -\frac{1}{2}\lambda_{\min}(Q)(1-b)\left|e_c\right|^2 \tag{40}$$

Therefore, from Lemma 1, the control error dynamic system (35) with regard to e_o is ISS, and:

$$|e_c| \leq \sqrt{\frac{\lambda_{\max}(N)}{\lambda_{\min}(N)}} \times \max\left(\left|e_c\left(t_0\right)\right| e^{-\gamma_2(t-t_0)}, \frac{|NB_u|\,|e_o|}{\lambda_{\min}(Q)b}\right), \forall t \geqslant t_0. \tag{41}$$

where $\gamma_2 = \frac{\lambda_{\min}(Q)}{\lambda_{\max}(N)}(1-b)$. □

3.3 Main Results

Theorem 1. *Consider that Assumptions 1–4 are satisfied, the observer gain matrix $\varepsilon_1, \varepsilon_2, \varepsilon_3$ and the control gain matrix k_1, k_2 are chosen properly,* (23) *and* (35) *are ISS. All the error signals are uniformly and ultimately bounded (UUB).*

Proof. From Lemmas 2 and 3, the estimation error dynamic system (23) and the control error dynamic system (35) are ISS as long as both A_o and A_c are Hurwitz.

$$|e_o| \leq \sqrt{\frac{\lambda_{\max}(M)}{\lambda_{\min}(M)}} \times \max\left(\left|e_o\left(t_0\right)\right| e^{-\gamma_1(t-t_0)}, \frac{4\overline{h}|M|}{\lambda_{\min}(P)a}\right), \forall t \geqslant t_0 \tag{42}$$

$$|e_c| \leq \sqrt{\frac{\lambda_{\max}(N)}{\lambda_{\min}(N)}} \times \max\left(\left|e_c\left(t_0\right)\right| e^{-\gamma_2(t-t_0)}, \frac{|NB_u|\,|e_o|}{\lambda_{\min}(Q)b}\right), \forall t \geqslant t_0 \tag{43}$$

As $t \to \infty$,

$$|e_o| \leq \sqrt{\frac{\lambda_{\max}(M)}{\lambda_{\min}(M)}}\left(\frac{4\overline{h}|M|}{\lambda_{\min}(P)a}\right) \tag{44}$$

$$|e_c| \leq \sqrt{\frac{\lambda_{\max}(N)}{\lambda_{\min}(N)}}\left(\frac{|NB_u|\sqrt{\frac{\lambda_{\max}(M)}{\lambda_{\min}(M)}}\left(\frac{4\overline{h}|M|}{\lambda_{\min}(P)a}\right)}{\lambda_{\min}(Q)b}\right) \tag{45}$$

Letting $e_r = q_{l1} - q_r = q_{l1a} - q_{l1r} = \begin{bmatrix} I & 0 & 0 \end{bmatrix} e_c$, From (35), one can get:

$$e_r = \begin{bmatrix} I & 0 & 0 \end{bmatrix}\left[A_c^{-1}\dot{e}_c - A_c^{-1}B_u e_o\right] \tag{46}$$

It shows that e_r is immune from the disturbance D. As $t \to \infty$, from(46), it yields:

$$|e_r| \le |e_c| = \sqrt{\frac{\lambda_{\max}(N)}{\lambda_{\min}(N)}} \left(\frac{|NB_u| \sqrt{\frac{\lambda_{\max}(M)}{\lambda_{\min}(M)}} \left(\frac{4\bar{h}|M|}{\lambda_{\min}(P)a} \right)}{\lambda_{\min}(Q)b} \right) \quad (47)$$

□

4. Simulation

In this part, the proposed output-feedback controller is demonstrated by a WMR trajectory-tracking system. The reference trajectory is given by:

$$\begin{cases} x_r = 5\cos t \\ y_r = 5\sin t \\ \vartheta_r = \arctan\left(\frac{\dot{y}_r}{\dot{x}_r}\right) \end{cases} \quad (48)$$

The initial values of WMR and the reference trajectory are fixed as $[310]^T$ and $[5\ 0\ \pi/2]^T$. The observer gain matrix $\varepsilon_1 = \mathrm{diag}\{-90,\ -90\}$, $\varepsilon_2 = \mathrm{diag}\{-2700,\ -2700\}$, and $\varepsilon_3 = \mathrm{diag}\{-27000,\ -27000\}$. The output feedback controller parameters are chosen as $k_1 = \mathrm{diag}\{103, 103\}$, $k_2 = \mathrm{diag}\{20, 20\}$. The external disturbance $\tau_{dr} = 1.5 \sin(t)$, $\tau_{dl} = -1.5 \sin(t)$. As shown in (7), with proper observer gains and controller gains, ν and ω are bounded. Thus, $d_{\tau r}$ and $d_{\tau l}$ are bounded and differentiable. The derivatives of $d_{\tau r}$ and $d_{\tau l}$ are bounded too. Assumption 2 is satisfied.

Figure 2 depicts that the WMR can follow the given trajectory regardless of the disturbance. The x, y, and φ tracking results in Fig. 3 show that the WMR can track the given trajectory fast without any overshoot. The tracking errors x_e, y_e, and φ_e are plotted in Fig. 4, and all the error signals converge to 0 in a short time. Different initial conditions are considered and the simulation results are shown in Fig. 5, where, $q_1(0) = \begin{bmatrix} -2 & 0 & 0 \end{bmatrix}^T$, $q_2(0) = \begin{bmatrix} 0 & -2 & \frac{\pi}{2} \end{bmatrix}^T$, $q_3(0) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$, $q_4(0) = \begin{bmatrix} 0 & 2 & -\frac{\pi}{2} \end{bmatrix}^T$, and $q_5(0) = \begin{bmatrix} 3 & 1 & 0 \end{bmatrix}^T$.

To show the superiority of the proposed method, an ESO based output-feedback controller is designed as a comparative. The results are given in Figs. 6–8. The controlled errors of x and y of the ESO based controller are greater than the proposed method. With the same observer gain matrix, the proposed control method can achieve more precise tracking than the ESO based control method. By introducing the favorable disturbance into EESO, EESO can take advantage of the active part of the disturbance. EESO has the same structure and follows the same design procedure as ESO. It just introduces the reference signal as correction compared with ESO. To achieve the same control performance, the ESO needs more energy than the EESO. The ESO-based controller will consume more computing resources.

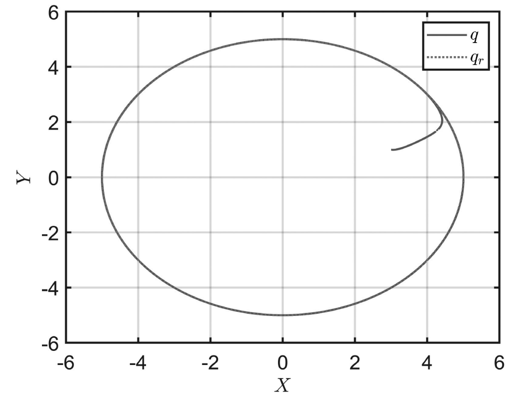

Figure 2. The trajectory of the proposed method.

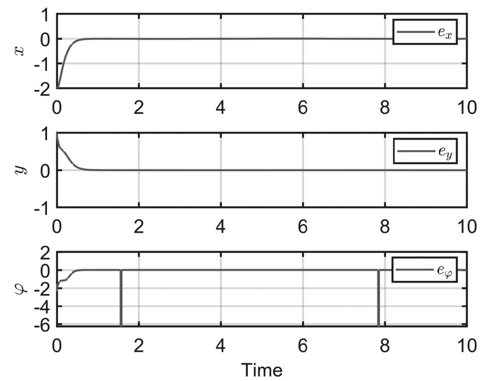

Figure 3. The tracking errors of the proposed method.

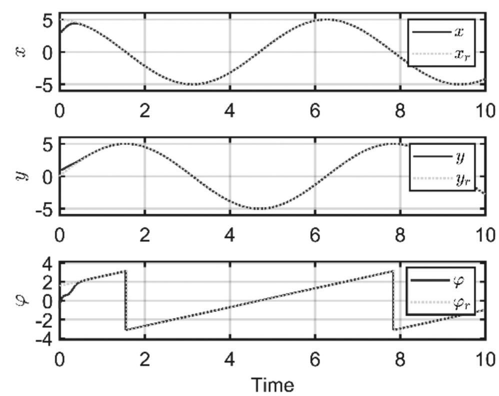

Figure 4. The tracking results of the proposed method.

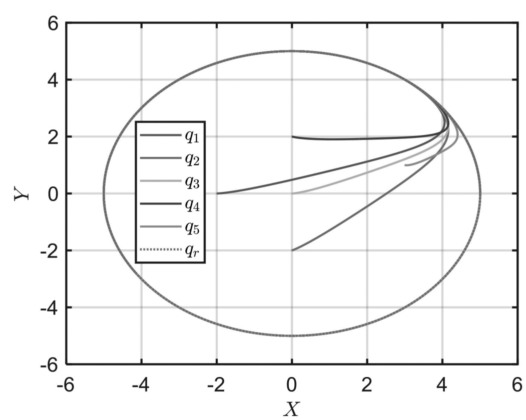

Figure 5. The tracking results of different initial conditions.

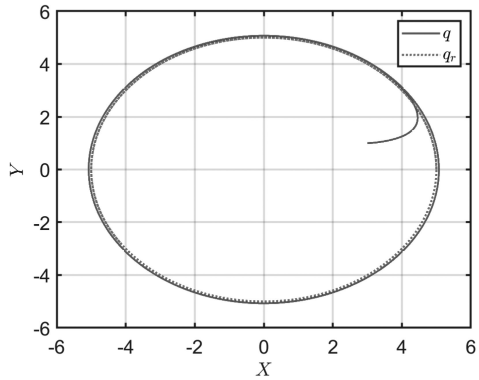

Figure 6. The trajectory of ESO based method.

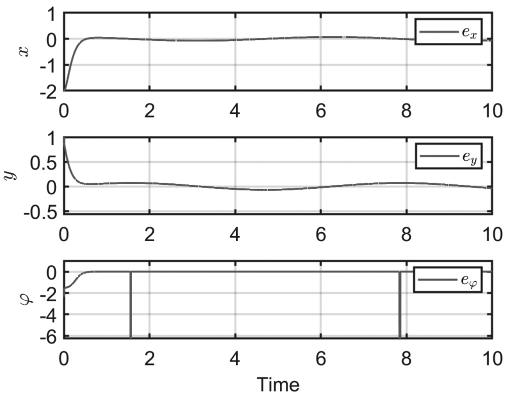

Figure 7. The tracking errors of ESO based method.

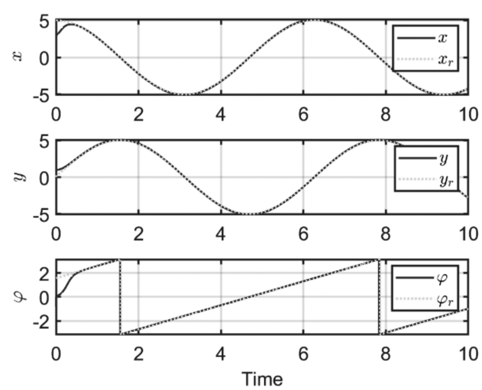

Figure 8. The tracking results of ESO based method.

5. Conclusion

Through the discussion of the function of disturbance for a perturbed trajectory tracking system, an actual system and the reference system are proposed in this paper. Meanwhile, considering that only position information x, y, and φ are available, the use of a state observer is necessary. By designing ESOs for these systems, the total disturbance, favorable disturbance, and unmeasured states are estimated. Then, an EESO is constructed to estimate the negative disturbance, which disturbs the stable tracking system. Thus, an output feedback controller is developed to compensate for the negative disturbance and ensure that tracking error e_r is uniformly and UUB. The simulation results verify that the trajectory tracking and disturbance rejection can be balanced.

To demonstrate the superior performance of the designed EESO in this manuscript, the controller is constructed as a relatively simple one. To achieve more precise tracking performance, many control strategies are proposed based on different convergence times of the tracking errors, such as finite-time control or prescribed performance control [25], [26]. The finite-time controller will be considered in future work.

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