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POWER FACTOR DECOMPOSITION DEPENDING ON THE TYPE OF DISTURBANCE

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ABSTRACT
It is well known that the power factor is the relation between active power and apparent power. Moreover, the square of the power factor represents the relationship between the minimum power loss and the present power loss. This paper deal with the decomposition of the latter ratio into three factors; the first related to the current imbalance, the second to the current distortion and third to the reactive power consumption. Once the contribution of each factor is known, it is posible to determine which disturbance produces the greatest losses. This concept is applied to a three phase-four wire system, with and without neutral resistance.

KEY WORDS
Power definitions, power quality, measurement, line losses, neutral wire.

1. Introduction

Since the nineteenth century, scientists have been attempting to assess the physical meaning of power flow in the nonsinusoidal regime, particularly with regards to the formulation of the reactive power. Steinmetz was the first author to address this subject, although the first theories on non active power for nonsinusoidal waveform were formulated between 1920 and 1930 by Budeanu and Fryze [1]. Taking into account that the generation and use of electrical energy in that period were associated to a sinusoidal periodic waveform, these two theories were not developed by other authors until 1970 when the use of electronic converters began to appear in power systems.
A large number of definitions have been proposed to characterize the so-called non active power in electrical systems under nonsinusoidal conditions. These can be classified in two groups; time domain based theories whose main objective is to implement non active power filters [2]; and frequency domain based theories developed mainly for measurement of the harmonic content [3].
As well as defining active and non active power, several studies also addressed apparent power and power factor. Although several proposals have been put forward, no consensus has been reached regarding the definition of these magnitudes [4-7]. The measurement of the non active power and the relation between its components are other areas that have received a great deal of attention.
Since the proliferation of nonlinear loads proliferation seems to be an unstoppable process in current electrical distribution systems, power flow theories have become increasingly important in applications such as compensation, identification of the harmonic load, distortion voltage reduction and instrumentation.
At the same time, the utilities are interested in ensuring that those clients who cause additional power losses in the electrical power system pay extra in their energy bill for this concept. The users, meanwhile, want the electricity companies to provide them with a reduced distortion voltage.
Recently, the IEEE Trial-Use Standard 1459 developed a system that is meant to solve some of the above problems and to allow the user to measure and design instrumentation for energy and power quantification [8].
Based on the definitions presented in [8], this paper focuses on the relation between typical disturbances in current power systems, and the line losses that they cause. In order to achieve this goal, a numerical example is simulated for two general cases; three phase-four wire system, with and without neutral resistance.

2. Classical Apparent Power Definitions In Three Phase Circuits

The calculation of the apparent power, in imbalanced three-phase circuits, can be performed by means of several definitions. The IEEE dictionary [9] contemplates the definition of several terms of power.

- The vector apparent power, is given by the expression:

$$S_V = \sqrt{\left(\sum_{k=a}^{c} P_k\right)^2 + \left(\sum_{k=a}^{c} Q_{bk}\right)^2 + \left(\sum_{k=a}^{c} D_k\right)^2} \qquad (1)$$

- The arithmetic apparent power, is the sum of the powers of each phase:

$$S_A = \sum_{k=a}^{c} \sqrt{P_k^2 + Q_{bk}^2 + D_k^2} \qquad (2)$$

P_k being the active power, Q_{bk}, D_k the reactive power and distortion power (Budeanu), respectively, corresponding to phase k.

- The rms apparent power is defined as:

$$S_F = \sum_{k=a}^{c} \sqrt{P_k^2 + Q_{Fk}^2} = \sum_{k=a}^{c} V_k \quad I_k \qquad (3)$$

V_k, I_k are the rms values of voltage and current, corresponding to phase k. Q_{Fk} is Fryze´s reactive power demanded by the phase k, thus including not only the reactive effect but also the distortion.

3. IEEE Definition

The Nonsinusoidal Situations IEEE Working Group [4], determines the parameters of any three-phase network from an equivalent single-phase system. They define the Effective Apparent Power as:

$$S_e = 3 \quad V_e \quad I_e \qquad (4)$$

V_e being the Effective Voltage and I_e the Effective Current.

4. Line Losses

The expression of the losses is determined in the supply line from the defined quantities. For a greater simplicity of the formulation, it is assumed that the four wires have the same resistance value, R. The loss power in the line, for a generic operation regime i, is:

$$P_{Li} = \left(I_{ai}^2 + I_{bi}^2 + I_{ci}^2 + I_{ni}^2 \right) R \qquad (5)$$

Therefore:

$$P_{Li} = 3 I_{ei}^2 R \qquad (6)$$

The minimum power loss in the line, for the same value of power transmitted to the load, has the value:

$$P_{Lmn} = \left(I_a'^2 + I_b'^2 + I_c'^2 \right) R \qquad (7)$$

I_a', I_b', I_c' being sinusoidal currents, that is to say, the condition of minimum losses implies that the feeding voltage is sinusoidal and balanced.

Therefore, the power consumed in the load, for the minimum loss regime, is:

$$P_1^+ = 3 V_1^+ I_1^+ \cos \varphi_1^+ \qquad (8)$$

where φ_1^+ the voltage-current phase shift of the positive-sequence component.

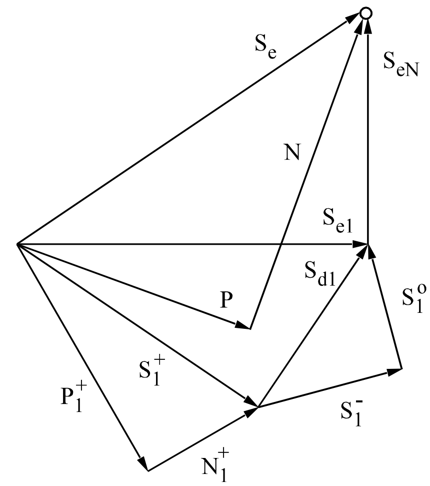

Figure1. Apparent power phasor diagram

From the Figure 1 it is possible to deduce another expression for the minimum power loss in the line:

$$P_{Lmn} = 3 I_1^{+2} \cos^2 \varphi_1^+ R \qquad (9)$$

Dividing (9) and (6) gives:

$$\frac{P_{Lmn}}{P_{Li}} = \frac{I_1^{+2} \cos^2 \varphi_1^+}{I_{ei}^2} \qquad (10)$$

and:

$$\frac{P_{Lmn}}{P_{Li}} = \left(\frac{S_1^+ \cos \varphi_1^+}{S_{ei}} \right)^2 \left(\frac{V_e}{V_1^+} \right)^2 \qquad (11)$$

$S_1^+ cos\varphi_1^+ / S_{ei}$ being the Positive-Sequence Effective Power Factor, P_{Fei}^+. Then:

$$\frac{P_{Lmn}}{P_{Li}} = \mathrm{P}_{\mathrm{Fe}i}^{+2} \left(\frac{V_e}{V_1^+} \right)^2 \qquad (12)$$

In most cases $P_{Fei}^+ \approx P_{Fei}$, $V_1^+ \approx V_e$, obtaining:

$$\frac{P_{Lmn}}{P_{Li}} \approx P_{Fei}^{\ 2} \qquad (13)$$

5. Power Factor Decomposition

It is possible to split the power factor of a power system into factors depending on different disturbances. This section describes a study of the different combinations of these disturbances. Case 0 is the reference situation: sinusoidal, balanced and with power factor equal to 1. The other three simple cases considered, are:

- Imbalanced current
- Non sinusoidal current

- Non null phase angle difference between voltage and current

The study is based on a three-phase circuit (Figure 2), with a null neutral impedance, switch *K* closed [10-12]. Table I shows the data related to these cases:

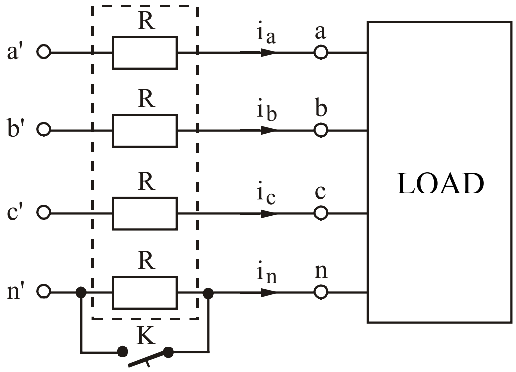

Figure 2. Three phase circuit model.

Nomenclature:

I_{im}: Imbalanced current
I_{ns}: Non sinusoidal current
P_i: Standardized power loss (case i)
R: Line resistance
P_{t0}: Consumed power (case 0)
P_{ti}: Consumed power (case i)
I_{ji}: Rms current-phase j (case i)
P_{Li}: Loss power (case i)
P_{Lmn}: Minimum loss power

Table I. Basic case quantities.

CASE	Voltage (V)	Current (A)	P_t (kW)
(0) Ideal	192.3	133.7	77.13
(1) I_{im}	192.3	I_a = 193.7 I_b = 133.7 I_c = 73.7	77.13
(2) I_{ns}	192.3	I_{a1}=134.9 I_{a7}=32.9	77.82
(3) $\varphi \neq 0°$	192.3	133.7 φ=30º(i)	66.80

The total consumed power in the rest of the cases will be different, so that the standardized losses in the line need to be determined for each of the assumptions and in reference to the power consumed in case 0, using the following expression:

$$P_i = R(I_{ai}^2 + I_{bi}^2 + I_{ci}^2)(P_{to} / P_{ti})^2 \tag{14}$$

Table II lists the vector apparent power, the effective apparent power, the standardized loss ratio and the disturbance type corresponding to the ideal case and the three simple ones.

Table II. Power and Loss Ratio

CASE	S_v (kVA)	S_e (kVA)	P_o / P_i	Disturbance
(0) Ideal	77.13	77.13	1	None
(1) I_{im}	77.13	82.15	0.882	I_-/I_+=0.2567 I_0/I_+=0.2619
(2) I_{ns}	80.12	80.12	0.943	THD_i=24.36%
(3) $\varphi \neq 0°$	77.13	77.13	0.750	PF_1=0.866

The next step is to investigate the different situations with more than one distortion type until completing the four remaining cases, giving all of the different possible combinations. The current imbalance is considered in case 1, the non-sinusoidal current constitutes case 2 and, for the reactive consumption, an inductive phase angle of 30º is applied in case 3. The effective and vector power factor, as well as the relation of losses in the line, is summarized in tables III and IV, where “1” indicates that the condition is not fulfilled and “0”, consequently, that the condition is fulfilled.

Table III. Conditions that are fulfilled in each one of the cases.

CASE	I_{im}	I_{ns}	$\varphi \neq 0°$
0	1	1	1
1	0	1	1
2	1	0	1
3	1	1	0
4	0	0	1
5	0	1	0
6	1	0	0
7	0	0	0

Table IV. Power factors and ratio of power losses in the line for the cases studied.

CASE	PF_v	PF_e	P_o/P_i	PF_{ei}^2
0	1	1	1	1
1	1	0.939	0.8816	0.8816
2	0.9715	0.9715	0.9439	0.9439
3	0.866	0.866	0.75	0.75
4	0.9715	0.9122	0.8321	0.8321
5	0.866	0.8132	0.6612	0.6612
6	0.8414	0.8414	0.7079	0.7079
7	0.8414	0.79	0.6241	0.6241

The square of the effective power factor PF_{ei}^2 and the ratio of power losses in the line P_o/P_i are equal in all the cases, which corroborates the case that points to PF_e as a suitable form of power factor, rather than the present one, since it includes the effects of the reactive demand, the presence of harmonics and the imbalanced network.

In a more detailed analysis of Tables III and IV, it is verified that the effective power factor, in any of the cases that do not meet more than one condition of the ideal circuit, could be obtained as the product of the corresponding P_{Fei} values. Thus in case 7, where it does not have any of the three qualities, the following equation is verified:

$$P_{Fe7}^{2} = P_{Fe1}^{2} \; P_{Fe2}^{2} \; P_{Fe3}^{2} \tag{15}$$

In the same way, it must be fulfilled that:

$$\frac{P_0}{P_7} = \left(\frac{P_0}{P_1}\right)\left(\frac{P_0}{P_2}\right)\left(\frac{P_0}{P_3}\right) \tag{16}$$

The above expressions show the main advantage of the effective power factor decomposition, since it reflects the incidence that each quality has on the loss of performance of the network. Thus, according to Table IV, the power loss ratio in case 7 is 0.6241 and can be split into three factors. The factor with the lowest incidence, 0.9439, is due to the current distortion, which has a moderate value, 24.36%. The current imbalance, whose ratio between negative and positive sequence components is 0.2567, has an influence of 0.8816. And finally, the inductive phase angle of 30º is the one that produces the greatest losses with a factor of 0.75.

Therefore, it has been shown that, from the point of view of power efficiency, the incidence of the harmonic content of the current is of less importance than that caused by the current imbalance or the reactive consumption, for the indicated reference values. Obviously, with other references, each disturbance would provide different line losses. However, the decomposition would inform us which distortion class is most detrimental in every particular example.

6. Evolution of Losses and Disturbances

In the previous point the switch *K* was closed, figure 1, now the three-phase system with switch *K* opened is also considered, so that to calculate the power losses it must take into account the neutral resistance, that it is supposed to be equal to the one of the phases, *R*.

$$P_i = R(I_{ai}^2 + I_{bi}^2 + I_{ci}^2 + I_n^2)(P_{to} / P_{ti})^2 \tag{17}$$

As a result of the previous modification two different behaviors arise, corresponding to the equations (10) and (13). This will influence the relationship between the losses in the line and disturbances. In order to undertake the study it has been considered a balanced sinusoidal three-phase system. Then, the currents imbalance level was increased in 14 steps , which was evaluated by the current ratio of inverse-direct sequence. In each one of the steps the two losses ratios are also calculated: P_o/P_i with *K* closed, and $(P_o/P_i)_n$ with *K* opened, Table V.

Table V. Correspondence between the imbalance degree and the power losses ratios.

Step	P_o/P_i	$(P_o/P_i)_n$	I_-/I_+
1	1.0000	1.0000	0
2	0.9505	0.8864	0.16145
3	0.9006	0.7825	0.2349
4	0.8505	0.6920	0.29644
5	0.8028	0.6161	0.35044
6	0.7522	0.5445	0.4059
7	0.7009	0.4800	0.46188
8	0.6514	0.4253	0.51731
9	0.6022	0.3758	0.57467
10	0.5535	0.3309	0.63506
11	0.5009	0.2864	0.70578
12	0.4521	0.2484	0.77847
13	0.4081	0.2160	0.85159
14	0.3557	0.1808	0.95172

As can be observed, the ratio $(P_o/P_i)_n$ is different from the ratio P_o/P_i, and this difference increase with the imbalance. In such a way that $(P_o/P_i)_n$ will not be able to be considered equal to the square of the power factor.

Afterwards, the calculation of the current distortion corresponding to each one of the previous steps is performed. The input of this process is the losses ratio P_o/P_i, and the rest of the stages are as follows:

The power factor is calculated on the corresponding losses ratio:

$$FP = \sqrt{\frac{P_o}{P_i}} \tag{18}$$

The equivalent apparent power is obtained from the active power in case 2, current distortion:

$$S_e = \frac{P_t}{FP} \tag{19}$$

The equivalent and harmonic currents are determined in order to know the current distortion:

$$I_e = \frac{S_e}{3\,V_e} \tag{20}$$

$$I_H = \sqrt{I_e^2 - I_1^2} \tag{21}$$

The fundamental current I1 has been considered constant and equal to the one of case 2. And finally the current distortion is given by:

$$THDi = \frac{I_H}{I_e} \tag{22}$$

Applying the previous calculation to the fourteen losses ratios, table V, the results shown in figure 3 are obtained. Three concepts are compared: current distortion, losses in the line, considering the neutral impedance (13) and without considering it (10), and finally, the current ratio of inverse and direct sequence.

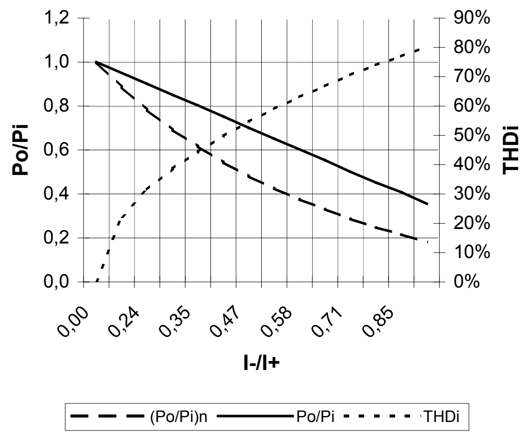

Figure 3. Losses in Line-Imbalance-Current Distortion

The difference between the losses ratio that considers the neutral impedance and the one that does not consider it, is quite small for reduced imbalances. This allows us to use the power factor decomposition presented like an approximated tool in three-phase four wire systems.

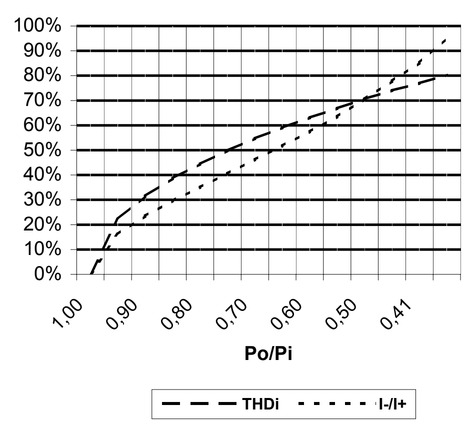

Figure 4. THDi, I-/i+ versus power losses in the line.

In figure 4, the evolution of current distortion and imbalance level versus power losses in the line, for the considered example, are shown. This way, the sensitivity that disturbances present when the losses increase, can be observed. So, the current distortion has a higher value than the imbalance until the losses ratio reaches a value of approximately 50%, the tendency reversing at that point.

The table VI shows some of the calculations made in order to study the relationship between the losses in the line and disturbances.

Table VI. Disturbances obtained for a progressive increase of the losses in the line

P_o/P_i	I_o/I_+	I_-/I_+	THD_i	FP_1
1.0000	0.00%	0.00%	0.00%	1.0000
0.9505	16.43%	15.92%	22.26%	0.9749
0.9006	23.38%	23.64%	31.53%	0.9490
0.8505	29.35%	29.97%	38.66%	0.9222
0.8028	34.63%	35.48%	44.41%	0.8960
0.7522	40.08%	41.12%	49.78%	0.8673
0.7009	45.59%	46.79%	54.69%	0.8372
0.6514	51.32%	52.16%	59.04%	0.8071
0.6022	57.20%	57.75%	63.07%	0.7760
0.5535	63.36%	63.67%	66.82%	0.7440
0.5009	70.56%	70.61%	70.64%	0.7078
0.4521	77.94%	77.77%	74.02%	0.6724
0.4081	85.10%	85.23%	76.94%	0.6388
0.3557	95.16%	95.19%	80.27%	0.5964

7. Conclusions

This research work focuses on the relation between the power losses in a line and three classic disturbances, namely current distortion, current imbalance and reactive consumption. It has been shown that in three-phase four-wire systems, without neutral resistance, it is posible to decompose the effective power factor into three subfactors depending on the influence of each disturbance.

Regarding the context of practical application, this new power factor approach is meant to be a useful tool at the time of defining a “fair” billing structure. In this way, each energy consumer will pay according to the disturbances he causes.

A three-phase four-wire system with neutral resistance was also studied and by comparing with the results obtained without neutral resistance, it is possible to appreciate that the difference between the two is not great, at least for reduced neutral currents.

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