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# A NEW APPROACH ON STATE IDENTIFICATION OF UNDERDETERMINDED MV GRIDS BASED ON GRID REDUCTION

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**ABSTRACT**
For historical reasons measurement facilities in distribution grids are sparsely spread and offer little information on grid state with moreover poor quality. Nowadays considering the increasing amount of distributed generation and the ambition of local utilities to decentralize energy management systems knowledge of grid state becomes more and more important. That is why algorithms for state approximation of underdetermined distribution grids are developed.

In this paper a new approach on state identification of underdetermined mean voltage grids based on grid reduction is introduced. The method particularly exploits typical distribution grid structures and already installed measurement facilities to improve data quality and reproduce lost information.
Grid reduction perfectly fits demands of identification methods based on sensitivity analysis and remarkably enhances grid state estimation.

Typically only magnitudes of voltages at central busbars and current magnitudes of connected lines are measured. By reducing the grid to these central busbars and some auxiliary nodes the entire complex-valued state of the reduced grid can be estimated. These results offer inference on condition of the whole original grid and can be used for other grid state identification methods like sensitivity analysis.

In this paper grid reduction and calculation of the reduced complex state is mainly focussed.

**KEY WORDS**
Energy management system, Metering, Distribution grid, Underdetermined power system, Grid state identification, Sensitivity analysis

## 1. Introduction

The increasing amount of distributed generation results in dramatically changed demands on mean (MV) and low voltage (LV) level grids originally planned as distribution grids with one way power flow direction from the HV/MV transformers to the end customers in the MV grid and in the LV grid [1], [2]. Nowadays load flow may change arbitrarily, mainly depending on fluctuations of supplied power of distributed sources and partly on the fluctuations of the load demand. Thus the power exchange between the superposed grid and the MV/LV layer can change in both directions.

The high variability especially of feeders based on renewable sources like wind energy results in fast changing, hardly determinable and possibly illegal grid states. So knowledge of grid state becomes more and more important. Unfortunately measurement technique is sparsely distributed in MV grids so observation vector is underdetermined in opposite to HV grids which are equipped with overdetermined observation vectors for conventional state estimation. Hence distribution grid state is not completely observable and has to be approximated. So research is done on algorithms for grid state identification.

Previous methods [3], [4] targeted on searching as few as possible new and valuable measurands to keep estimation error within a predefined tolerance and investment cost on an acceptable ratio. In contrast grid reduction tries to more intensely exploit existent measurands to accurately determine complex values of voltage and currents at central busbars of the distribution grid.

The gained information may be used on the ones hand to support previous grid state identification methods i.e. methods based on sensitivity analysis. On the other hand methods to back-reference to the original grid are about to be developed.

## 2. Description of measurement situation

Mean (10 or 20 kV) and low (0.4 kV) voltage grids are summarised as distribution grids. Mostly these grids are constructed as cable networks due to higher transmissibility, protection of the countryside and less susceptibility to atmospheric disturbances. To keep investment costs as low as possible topology of MV grids is kept simple. Typically these grids are planned as radial grids or interrupted single-mesh-grids with bilateral supply. This benefits cheap and simple grid protection [5].

As mentioned in the introduction measurement technique is sparsely spread in MV grids. Nevertheless at least active and reactive power flows at hand-over points and voltages of central bus bars are measured. Terminal current RMS at these bus bars are often measured as well to locate faults and to have a rough estimation of line utilisation. All needed values can be obtained by cheap measurement facilities with low accuracy. Unfortunately a lot of information like phase angles or even power flow direction is lost so data quality is poor.

Fig. 1 shows the structure of a small MV grid which is used for demonstration purposes. As mentioned above distribution grids are operated with open meshes. In this case all switches are closed to show the worst cases considering state identification due to a radial grid structure would have simplified estimation of grid condition.

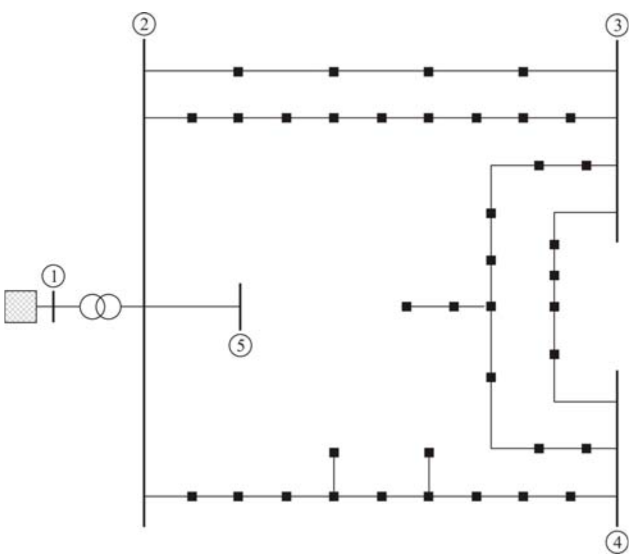

Figure 1: demonstration grid

Nodes 1 to 5 are supposed to be observed bus bars with available information on voltage magnitudes. Furthermore current magnitudes at terminals connected to the five highlighted nodes are measured. There is no information of voltage, current or power on all other nodes (black squares) available.

## 3. Grid reduction algorithm

The goal of grid state identification is the best estimation of grid condition using as few additional measurands as possible. Grid reduction offers a way to reconstruct lost information on voltage and currents angles at bus bars to support grid state identification. Thus there is no need of installing new devices.

Grid reduction can be divided into three consecutive steps which are shown in the following subchapters and are demonstrated using the given demonstration grid.

### 3.1 Grid simplification

The first step is to eliminate branch lines with unobserved nodes at both ends such that the whole grid now only contains tie-lines starting and ending at observed bus bars. The example grid is reduced as shown in fig. 2. Nominal powers of removed nodes are added to the remaining node on the line the branch line previously was connected to. This indeed results in a little error due to grid losses on the branch line but the error is negligible.

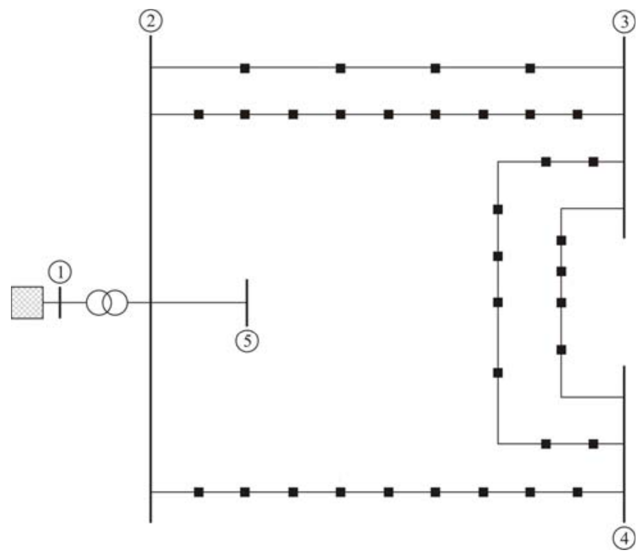

Figure 2: demonstration grid after grid simplification

### 3.2 Reduction of nodes

In the next step all nodes on each tie-line are removed and replaced by a single node at a certain position on the line. To obtain this position the parameters *r, x, c, g* and the length *l* of the complete line have to be calculated from the parameters of all subsections.

A forecast or presumption of a nodal power or current vector ***f*** is also needed to obtain the optimal position of the auxiliary node on the line. If there is no information on load type mixture or power time series available rated powers of transformers feeding the LV grids are effectual due to error induced by non-optimal location is relatively small. The position of the substitution node is calculated using the rule of torque [6], [7].

$$l_{\mathrm{Sub}} = \frac{\sum\limits_{n=\mathrm{n_{min}}}^{\mathrm{n_{max}}} F_n \, d_n}{\sum\limits_{n=\mathrm{n_{min}}}^{\mathrm{n_{max}}} F_n} \tag{1}$$

$d_n$ is the distance between node $n$ and the observed bus bar with the smaller number and $\boldsymbol{f}$ includes power or current injection into the tie-line.

The complete tie-line is now divided by the substitution node into two parts with parameters

$$r_1 = r\frac{l_1}{l},\, x_1 = x\frac{l_1}{l},\, c_1 = c\frac{l_1}{l},\, g_1 = g\frac{l_1}{l} \tag{2}$$

and

$$r_2 = r\frac{l-l_1}{l},\, x_2 = x\frac{l-l_1}{l},\, c_2 = c\frac{l-l_1}{l},\, g_2 = g\frac{l-l_1}{l} \tag{3}$$

The demonstration grid now is fully reduced as to been seen in fig. 3 and contains only all observed bus bars and tie-lines with one auxiliary node. Quadrupoles which are observed at both sides (the transformer with terminals 1 and 2 and the line with terminals 23 and 24 in fig. 3) do not need to be split up with an additional node.

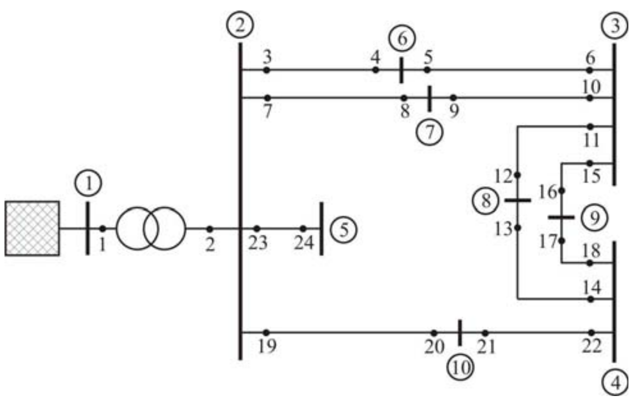


Figure 3: fully reduced demonstration grid

Note that the nodal power assumption is only needed to find an adequate position of the replacement node and has no influence on subsequent steps.

There are some other ways to define the node position. For example it is possible as well to put it in the electric middle of the tie-line or find a position according to

$$\frac{l_1}{l-l_1} = \frac{I_{\mathrm{T,nmax}}}{I_{\mathrm{T,nmin}}} \tag{4}$$

Previous investigations have proven that the suggested method (rule of torque) provides the best solution and smallest error.

### 3.3 Reconstruction of complex current and voltage values at observed bus bars

Unknown values are

- voltage magnitudes at auxiliary nodes
- all voltage angles except at the slack node
- terminal current magnitudes at auxiliary node
- all terminal current angles

Nodal currents at auxiliary nodes are not used as independent variable since they can be calculated from terminal currents

$$\underline{\boldsymbol{i}}_{\mathrm{K}} = \boldsymbol{K}_{\mathrm{KT}} \, \underline{\boldsymbol{i}}_{\mathrm{T}} \tag{5}$$

using nodal-terminal-incidence matrix $\boldsymbol{K}_{\mathrm{KT}}$.

It is obvious that variables can be divided into two groups: nodal and terminal variables. Both groups are combined to a state vector $\boldsymbol{x}_{\mathrm{red}}$ of independent variables

$$\boldsymbol{x}_{\mathrm{red}} = \begin{bmatrix} \boldsymbol{u}_{\mathrm{K,u}} \\ \hline \boldsymbol{\delta}_{\mathrm{K,u}} \\ \hline \boldsymbol{i}_{\mathrm{T,u}} \\ \hline \boldsymbol{\varphi}_{\mathrm{T,u}} \end{bmatrix} \tag{6}$$

$\boldsymbol{x}_{\mathrm{red}}$ is completed by observed values such that

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{x}_{\mathrm{u}} \\ \hline \boldsymbol{x}_{\delta} \\ \hline \boldsymbol{x}_{\mathrm{i}} \\ \hline \boldsymbol{x}_{\varphi} \end{bmatrix} \tag{7}$$

with

$$\boldsymbol{x}_{\mathrm{u}} = \begin{bmatrix} \boldsymbol{u}_{\mathrm{obs}} \\ \boldsymbol{u}_{\mathrm{K,u}} \end{bmatrix}, \quad \boldsymbol{x}_{\delta} = \begin{bmatrix} 0 \\ \boldsymbol{\delta}_{\mathrm{K,u}} \end{bmatrix} \tag{8}$$

and

$$\boldsymbol{x}_{\mathrm{i}} = \begin{bmatrix} \boldsymbol{i}_{\mathrm{obs}} \\ \boldsymbol{i}_{\mathrm{T,u}} \end{bmatrix}, \quad \boldsymbol{x}_{\varphi} = \boldsymbol{\varphi}_{\mathrm{T,u}} \tag{9}$$

Using the elements of $\boldsymbol{x}$ it is possible to calculate complex nodal voltages and terminal currents

$$\underline{\boldsymbol{u}}_{\mathrm{K}} = \mathrm{diag}\left(\boldsymbol{x}_{\mathrm{u}}\right) \mathrm{e}^{\mathrm{j}\,\boldsymbol{x}_{\delta}} \tag{10}$$

and

$$\underline{\boldsymbol{i}}_{\mathrm{T}} = \mathrm{diag}\left(\boldsymbol{x}_{\mathrm{i}}\right) \mathrm{e}^{\mathrm{j}\,\boldsymbol{x}_{\varphi}} \tag{11}$$

To grant a better overview, two auxiliary vectors $\boldsymbol{m}_{\mathrm{K}}$ containing the numbers of observed nodes and $\boldsymbol{m}_{\mathrm{T}}$ containing the numbers of observed terminals are introduced.

According to Kirchhoff's law the equations

$$\underline{\boldsymbol{i}}_{\mathrm{K,obs}} + \boldsymbol{K}_{\mathrm{KT}}\left(\boldsymbol{m}_{\mathrm{T}},:\right)\underline{\boldsymbol{i}}_{\mathrm{T}} = \mathbf{0} \tag{12}$$

and

$$\underline{\boldsymbol{i}}_{\mathrm{K,obs}} - \underline{\boldsymbol{Y}}_{\mathrm{KK}}\left(\boldsymbol{m}_{\mathrm{K}},:\right)\underline{\boldsymbol{u}}_{\mathrm{K}} = \mathbf{0} \tag{13}$$

have to be fulfilled. Unfortunately this equation system still is underdetermined. But as $\boldsymbol{x}$ contains two groups of variables each with tie ability to exactly define the grid state it is mandatory to match both states resulting in two additional equations

$$\underline{\boldsymbol{i}}_{\mathrm{T}} - \underline{\boldsymbol{Y}}_{\mathrm{T}}\,\boldsymbol{K}_{\mathrm{KT}}^{\mathrm{T}}\,\underline{\boldsymbol{u}}_{\mathrm{K}} = \mathbf{0} \tag{14}$$

and

$$\underline{\boldsymbol{u}}_{\mathrm{K}} - \underline{\boldsymbol{u}}_{\mathrm{K}}\left(\underline{\boldsymbol{i}}_{\mathrm{T}}\right) = \mathbf{0} \tag{15}$$

with

$$\underline{\boldsymbol{u}}_{\mathrm{K}}\left(\underline{\boldsymbol{i}}_{\mathrm{T}}\right) = \begin{bmatrix} \underline{U}_{\mathrm{slack}} \\ \underline{\boldsymbol{Y}}_{\mathrm{KK,sred}}^{-1}\left(\boldsymbol{K}_{\mathrm{KT}}\,\underline{\boldsymbol{i}}_{\mathrm{T}} - \underline{\boldsymbol{y}}_{\mathrm{KK,s}}\,\underline{U}_{\mathrm{slack}}\right) \end{bmatrix} \tag{16}$$

Eq. 12 to 15 now form an overdetermined system which can be solved with any least-squares method adequate to conventional power system state estimation [8].

### 3.4 Reflection on Accuracy

The method is able to very accurately reproduce the complex voltages at central busbars due to voltage magnitudes are nearly equal.

Phase angle accuracy of terminal currents depends on current magnitudes which are used as weighting factors at the least-squares algorithm. A small magnitude may result in a higher phase angle deviation but reduces error influence as well.

### 3.5 Field of application

Grid state identification by sensitivity analysis uses existing real-valued measurands and knowledge of their influence on any nodal voltage to approximate a state vector. In association with grid reduction it is possible to offer complex-valued measurands and thereby double the number of influencing variables which reduces approximation error without installing new measurement facilities and thus saving investment cost.

On the other hand de-reduction methods are now being developed to directly infer complex-valued state of the entire grid from the reduced one.

## 4. Example

A test scenario with a predefined nodal power vector is used to demonstrate functionality of grid reduction algorithm using the demonstration grid shown in fig. 1.
A power flow calculation returns a consistent set of complex nodal voltages and terminal currents to compare with the results obtained by grid state identification. Only magnitudes of voltage and current at the above mentioned busbars are given to the reduction algorithm.

The deviation of solutions is shown in the following tables.

Table 1: Deviation of nodal voltage results

| | $\Delta U$ | $\Delta\delta$ |
|---|---|---|
| Node 1 | 0 kV | 0 deg |
| Node 2 | 0 kV | 0.0009 deg |
| Node 3 | 0 kV | 0.0047 deg |
| Node 4 | 0 kV | 0.0086 deg |
| Node 5 | 0 kV | 0.0009 deg |

Table 2: Deviation of terminal current results

| | $\Delta I$ | $\Delta\varphi$ |
|---|---|---|
| Terminal 1 | 0 A | -0.0003 deg |
| Terminal 2 | 0 A | 0.0006 deg |
| Terminal 3 | 0 A | 0.1024 deg |
| Terminal 4 | 0 A | -0.9303 deg |
| Terminal 5 | 0 A | 0.0802 deg |
| Terminal 6 | 0 A | -2.5297 deg |
| Terminal 7 | 0 A | -3.4287 deg |
| Terminal 8 | 0 A | -2.7949 deg |
| Terminal 9 | 0 A | 1.9105 deg |
| Terminal 10 | 0 A | -15.1632 deg |
| Terminal 11 | 0 A | -0.1589 deg |
| Terminal 12 | 0 A | -0.7709 deg |
| Terminal 13 | 0 A | 0.0177 deg |
| Terminal 14 | 0 A | 0.0076 deg |

Deviation of current angle is unusual high due to current magnitude at terminal 10 is close to zero so this has no effect on accuracy as mentioned in chapter 3.4.

## 5. Conclusion

State identification of MV grids based on a reduction algorithm is a fast, stable und reliable way to nearly faultlessly obtain voltages, currents and phase angles at central busbars of mean voltage grids without installation of new measurement facilities but only using existent sources of information.

The methods reduces the number of nodes and therewith the number of variables and uses two concurrent state vectors to obtain an overdetermined equation system from an underdetermined one which can be solved by least-squares-algorithms. Approximation accuracy especially of central busbar voltages is higher than estimation by former identification methods.

Results of grid reduction are used to support identification methods based on sensitivity analysis which are needed in proximate steps to estimate voltages at all other nodes. Combination of both methods should result in even more accurate approximations which will be investigated during further research.
Additionally methods to directly infer condition of the entire grid are developed.

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