

## **A NOVEL STATE ESTIMATION TECHNIQUE FOR IDENTIFICATION OF TRANSFORMER INRUSH CURRENTS**

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### **ABSTRACT**

This paper presents a new method to identify the inrush and fault currents for transformer protection. When a power transformer is energized, the resulting inrush current can cause a false tripping of protection system. The second harmonic contents of the inrush current is used to differentiate between the inrush and the fault situations. Estimation of the contents of the current signal is achieved using a fast and new algorithm. The main advantage of the proposed technique is its ability in producing the estimates in a very short time and at a very high degree of accuracy. The algorithm uses sets of digital samples of the distorted waveforms of the inrush and fault currents to extract the features of the signals. A simple expert system is then used to evaluate the results in order to differentiate between different situations. The proposed technique has been tested using simulated power system. The extensive simulation studies have indicated that the restrain signal can be issued in less than a quarter of the cycle, therefore the method can be used as an effective tool for high speed digital relaying.

### **KEYWORDS**

Transformer inrush, harmonics, estimation, protection.

### **1. Introduction**

The power transformer is an important component of power system for which various protective schemes have been developed. Any power transformer protective scheme has to take into account the effect of magnetization inrush currents. Based on the switching angles, magnitude and polarity of the remnant flux, the magnitude of the inrush current may reach 8-10 times of the full load current magnitude, causing a false operation of the protective system to happen [1]. So the kernel of the protection system is how to differentiate between the inrush current and fault currents. Accurate discrimination between the inrush current and fault currents has long been recognized as a very challenging problem to power transformer protection engineers.

There are many existing schemes proposed to cope with this problem. One of the most commonly used technique used to prevent false operation due to inrush current, is the harmonic restrain method. In these types of

relays, the operation of the relay is restrained or blocked under the inrush current conditions by monitoring the level of the second harmonic content of the measured current (some times the fifth in addition). Based on that, the second harmonic component of the inrush current is considerably larger than that in a typical fault current. A restrain signal is issued if the second harmonic content of the differential current exceeds a pre-defined value[2]. However, the main problem associated with the use of this approach is the estimation tool. Most of the technique used for estimating the harmonic contents needs from half to one complete cycle of short circuit current data. A fast action is needed to restrain the relay and prevent the false operation [1], [3].

Recently, many new techniques have been proposed to deal with forgoing problems in power transformer protection. Some of these techniques uses voltage variations at transformer terminals. Other techniques make use of the data obtained from both current and voltage acceleration [4] [5]. In addition, methods based on artificial intelligence techniques, have been also proposed. Most of these proposed methods have used the artificial neural networks (ANN) and fuzzy logic (FL) techniques [6], [7]. ANN needs Hugh amount of data for training. Also, the computational time that is needed for execution is still higher than some other methods. For the FL based techniques, there are still no recommended criteria for setting the internal parameters of a relay and the technique is not robust to deal with many commonly encountered transient conditions [3]. In addition to artificial intelligence techniques, wavelet transform (WT) has been applied in many recent researches [1], [3], [8].

This paper presents a novel digital based filtering technique for discrimination between an internal fault and other operating conditions, of power transformer, such as the magnetization inrush current and CT saturation. The proposed algorithm is a dynamic estimator based on stochastic estimation theorem, which is applicable for estimating and tracking of non-stationary signals[9], [10]. The samples obtained from the CT secondary winding are used to calculate some judgment indices. These indices can easily differentiate between the inter faults and other operating conditions in a very short time.

## 2. Mathematical formulation

This section presents the formulation needed for estimating the parameters of inrush and short circuit currents. These parameters namely are the DC, fundamental and harmonic magnitudes as well as the exponential decaying time constant if exists. The general form of the power transformer primary current is expressed as follows [1].

$$i(t) = Ae^{-c_j t} + \sum_{j=1}^N B_j \sin(\omega_j t + \theta_j) \quad (1)$$

where

A is the DC amplitude

j =1 for the fundamental component and 2,3,... for the harmonics

$\omega_j$  is the fundamental or harmonic frequency

$B_j$  is the amplitude of component j

$\theta_j$  is the phase angle of component j

$c_j$  is the inverse of the decaying time constant of the DC component

N is the highest order of harmonic considered in the problem.

Indeed during the saturation-free portion, the current transformer (CT) secondary current  $i_s(t)$  is an image of the primary current  $i(t)$  divided by the current transformer turns ratio.

Now, considering N harmonics, then equation 1 becomes

$$i(t) = Ae^{-c_1 t} + B_1 \sin(\omega_1 t + \theta_1) + B_2 \sin(\omega_2 t + \theta_2), \dots + B_N \sin(\omega_N t + \theta_N) \quad (2)$$

$$i(t) = A \left\{ 1 - tc + 0.5t^2 c^2 - t^3 c^3 / 6 \right\} + \dots + B_1 \left\{ \sin(\omega_1 t) \cos \theta_1 + \cos(\omega_1 t) \sin \theta_1 \right\} + \dots + B_2 \left\{ \sin(\omega_2 t) \cos \theta_2 + \cos(\omega_2 t) \sin \theta_2 \right\} + \dots + B_N \left\{ \sin(\omega_N t) \cos \theta_N + \cos(\omega_N t) \sin \theta_N \right\} \quad (3)$$

$$i(t) = H(1,1)X_1 + H(1,2)X_2 + H(1,3)X_3 + H(1,4)X_4 + H(1,5)X_5 + H(1,6)X_6 + \dots + H(1,2N+4)X_{2N+4} \quad (4)$$

This is done by using the first three terms of the exponential function expansion as well as using trigonometric (sin-cos) manipulation.

$$\begin{aligned} H_{(1,1)} &= 1 \\ H_{(1,2)} &= -t \\ H_{(1,3)} &= 0.5t^2 \\ H_{(1,4)} &= -\frac{t^3}{6} \\ H_{(1,2N+3)} &= \sin \omega_N t \\ H_{(1,2N+4)} &= \cos \omega_N t \end{aligned} \quad (5)$$

and

$$\begin{aligned} X_1 &= A \\ X_2 &= AC \\ X_3 &= AC^2 \\ X_4 &= AC^3 \\ X_{2N+3} &= B_N \cos \theta_N \\ X_{2N+4} &= B_N \sin \theta_N \end{aligned} \quad (6)$$

If the signal is sampled at a pre-selected rate,  $\Delta T$ , then m samples, for the signal would be obtained at  $t_1, t_2, \dots, t_m$ .

$$\begin{bmatrix} i(t_1) \\ i(t_2) \\ \dots \\ \dots \\ i(t_m) \end{bmatrix} = \begin{bmatrix} H_{(1,1)}(t_1) & H_{(1,2)}(t_1) & \dots & \dots & H_{(1,N)}(t_1) \\ H_{(2,1)}(t_2) & H_{(2,2)}(t_2) & \dots & \dots & H_{(2,N)}(t_2) \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ H_{(m,1)}(t_m) & H_{(m,2)}(t_m) & \dots & \dots & H_{(m,2N+4)}(t_m) \end{bmatrix} \begin{bmatrix} X_1(t) \\ X_2(t) \\ \dots \\ \dots \\ X_{2N+4}(t) \end{bmatrix} + \begin{bmatrix} e_1(t) \\ e_2(t) \\ \dots \\ \dots \\ e_{2N+4}(t) \end{bmatrix} \quad (7)$$

Now define  $u$  as the number of unknowns ( $u=2N+4b$ ) then we can write this system in the conventional discrete state space compact form as:

$$i(k) = H(k)X(k) + e(k) \quad (8)$$

where

$k$  is the discrete step number

$i(k)$   $m * 1$  measurement vector

$H(k)$   $m * u$  connection matrix

$X(k)$   $u * 1$  state vector to be estimated

$e(k)$   $u * 1$  error vector to be minimized

Once the state vector for the waveform is identified, the values of problem parameters can be calculated as

$$A = X_1, \quad C = \frac{X_2}{X_1} = \frac{X_3}{X_2} = \sqrt{\frac{X_3}{X_1}}$$

$$B_1 = \sqrt{X_6^2 + X_5^2}, \quad \theta_1 = \tan^{-1} \left( \frac{X_6}{X_5} \right)$$

$$B_2 = \sqrt{X_8^2 + X_7^2}, \quad \theta_2 = \tan^{-1} \left( \frac{X_8}{X_7} \right) \quad (9)$$

$$B_N = \sqrt{X_{2N+4}^2 + X_{2N+3}^2}, \quad \theta_N = \tan^{-1} \left( \frac{X_{2N+4}}{X_{2N+3}} \right)$$

### 3. Description of the proposed algorithm

The proposed algorithm consists of two main parts. In the first part, the waveform contents of equation 9 are estimated. Inrush or fault situations are identified and corresponding action signals are issued and sent to the relay based on evaluating three factors using a simple expert system.

#### 3.1 The Digital dynamic filter

In the first part, the on-line estimation process of the parameters described in section 2 is performed using the discrete least absolute value filtering algorithm (DLAVF). Although the complete derivation of the proposed filter equations is beyond the scope of this paper and given in reference [9], a short description is given next. The dynamic filter works on the discrete state space model described by the measurement equation and the state transition equation in the following form.

$$i(k) = H(k)X(k) + e(k) \quad (10)$$

$$X(k+1) = \Phi(k)X(k) + \varpi(k) \quad (11)$$

The state transition formulation depends on the type of reference chosen. Either stationary reference or rotating reference can be used [9]. The measurement error vector  $e(k)$  and the state error  $\varpi(k)$  are assumed to be white sequence with known covariance as,

$$E\{e(k)e(j)^T\} = \begin{cases} 0 & ; j \neq k \\ R(k) & ; j = k \end{cases} \quad (12)$$

$$E\{\varpi(k)\varpi(j)^T\} = \begin{cases} 0 & ; j \neq k \\ Q(k) & ; j = k \end{cases} \quad (13)$$

The initial condition of  $X(0)$  is a Gaussian random vector with the following statistics,

$$E\{X(0)\} = \bar{X}(0) \quad (14)$$

$$E\{[X(0) - \bar{X}(0)][X(0) - \bar{X}(0)]^T\} = \bar{P}(0) \quad (15)$$

where  $\bar{P}(0)$  is the initial error covariance matrix of the states, with dimensions  $u \times u$ . The covariance of the error

at any step ( $k$ ) can be obtained by replacing  $X(0)$  with  $X(k)$  in equation (17).

The algorithm starts with an initial estimate for the system parameter vector  $\bar{X}(0)$  and its error covariance matrix ( $\bar{P}(0)$ ) at some point  $k=0$ . These estimates are denoted as  $\bar{X}, \bar{P}$ , where  $(-)$  means that these are the best estimations at this point, prior to assimilating the measurement at instant  $k$ . With such initial values, of both parameters and error co-variances, filter gain matrix  $K(k)$  at this step is calculated as follows,

$$K(k) = \left[ H(k) + R(k)Ly^T \bar{P}^{-1}(k) \right]^{-1} \quad (16)$$

Assuming that the state vector dimension is  $u \times 1$ , the vectors  $L$  and  $y$  are defined as:  $L$  is  $u \times 1$  column vector  $(1,1,\dots,1)^T$ ; and  $y^T$  is  $1 \times u$  row vector  $(1,1)$  [9]. Using the filter gains, estimates are updated with measurements  $Z(k)$  through equation (20), and error co-variances for update estimates are computed from equation (19).

$$\hat{X}(k) = \bar{X}(k) + K(k)\{i(k) - H(k)\bar{X}(k)\} \quad (17)$$

$$P(k) = [I - K(k)H(k)]P(k)[I - K(k)H(k)]^T + K(k)R(k)K^T(K) \quad (18)$$

Finally, error co-variances and estimates are projected ahead to repeat with  $k=2$ .

$$\bar{P}(k+1) = \Phi(k)P(k)\Phi^T(k) + Q(k) \quad (19)$$

$$\bar{X}(k+1) = \Phi(k)\hat{X}(k) + R(k) \quad (20)$$

The process is repeated until the last sample is reached. It is assumed that the co-variances and the transition matrices are known.

#### 3.2 Detection

After analyzing the current signal, and obtaining the signal parameters (magnitudes, phase angles of the fundamental and harmonics, DC level and its time constant) the algorithm goes to the second part. In the second part of the algorithm, the detection of the inrush situation is performed.

The criterion used is the commonly used ratio between the second harmonic and the fundamental components. This characteristic is very important. However, to overcome the drawbacks associated with the use this criterion alone, another important characteristics of the inrush current are employed. Therefore, the fundamental component, fifth harmonic component and total harmonic distortion factor are used as a support criterion. The fundamental current value is found to be much higher in the fault cases than that of the inrush cases. This method is relatively easy to apply in a digital

relay because extraction of the desired components is performed using one digital filter.

#### 4. System Under Study

A practical power system is simulated to demonstrate the ability of the proposed filter to track the signal contents during normal and abnormal conditions in the power system. The system is simulated using Simulink and SimPower Systems block set. The simulated system is shown in Figure 1. A 450 MW, 500/230 kV power transformer is connected between two busses in the power system. The current transformers located at both sides of the transformer, are conventionally connected between line currents on each side of the power transformer. The sampling frequency used is 6 kHz. Different disturbances are simulated on this transformer including switching at no-load and load conditions, inter fault and other switching events. The resultant samples for each case are passed to the algorithm in order to calculate and evaluate the performance indices.

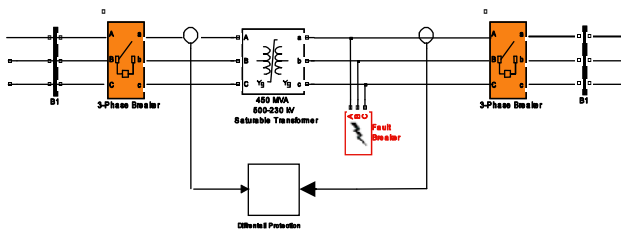


Figure 1 Study system

#### 5. Testing

The study system described in section 4 is used to test the performance of the proposed method. Tables 1 and 2 summarizes the study cases. Different operating conditions such as inrush currents, internal faults, with and without inrush and capacitor switching, are considered. These study cases are arranged in different groups as follows:

##### 5.1 Group#1

In this group, switching on a loaded transformer at different switching angles and at different remnant flux magnitudes are considered. Assume that the rated magnetic flux equals 1 p.u. different study cases are generated to cover the effects of varying both of the remnant flux and the switching on angle. Table 2 summarizes the considered cases while figures 2 and 3 give sample of the obtained waveforms. These figures correspond to cases 3 and 15. In these cases the remnant flux is 0.25 and 1.25 respectively as seen from table 1. It is clear that the maximum recorded current at first peak is much higher in case 15 than that in case 1.

Table 1  
Different Inrush Cases

Case #	Switching angle	Remnant flux (P.U.)
1,2,3	0°, 45°, 90°	0.25
4,5,6	0°, 45°, 90°	0.5
7,8,9	0°, 45°, 90°	0.75
10,11,12	0°, 45°, 90°	1.00
13,14,15	0°, 45°, 90°	1.25

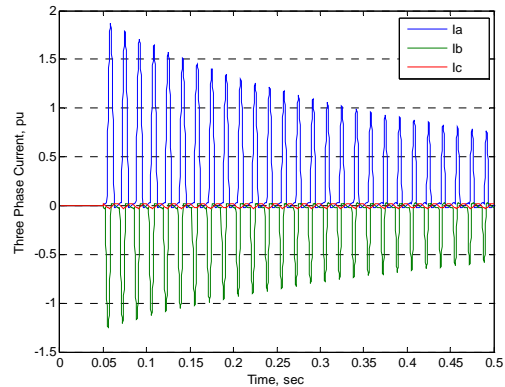


Figure 2 Three phase inrush currents , switching angle=90°, remnant flux=0.25

##### 5.2 Group#2

In group 2, different internal faults are considered. The fault inception angle is varied as well as the fault type. Based on the decided fault inception angle, the fault switching instant in seconds, is determined. Table 2 summarizes the considered cases in this group. Samples of the obtained waveforms are displayed in figures 4 and 5. These figures are for cases 16 and 20 respectively. The transformer is switched on at t=0.05 sec., then based on the fault angle, the a L-G fault is applied.

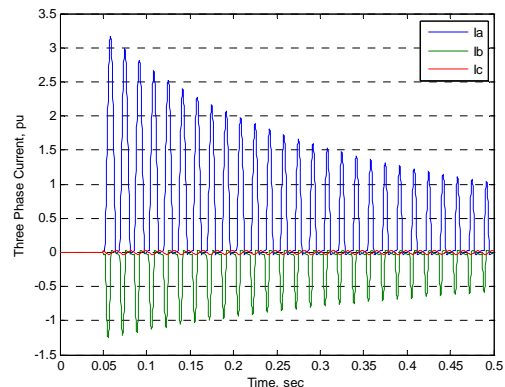


Figure 3 Three phase inrush currents, switching angle=90°, remnant flux=1.25

Table 2  
Different Fault Cases

Case study #	Fault inception angle	Type of fault
16-20	0°, 15°, 30°, 45°, 90°	L-G
21-25	0°, 15°, 30°, 45°, 90°	L-L
26-30	0°, 15°, 30°, 45°, 90°	L-L-G
31-35	0°, 15°, 30°, 45°, 90°	3-Phase

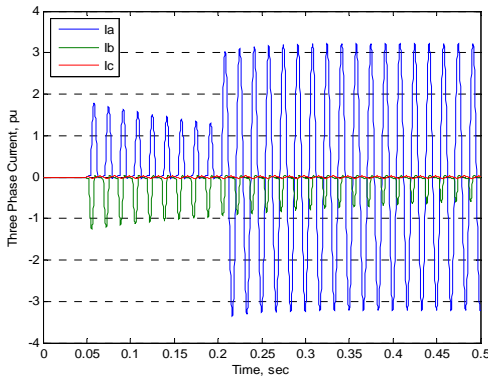


Figure 4 Three phase inrush currents and three phase fault currents (L-G), fault angle=0°

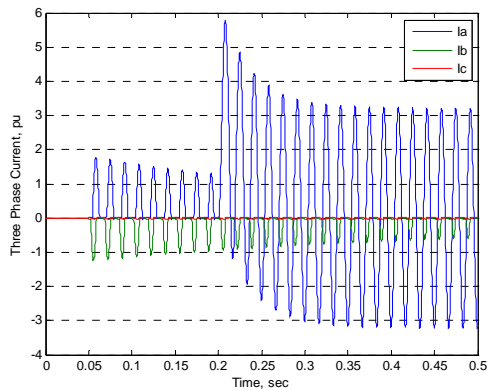


Figure 5 Three phase inrush currents and three phase fault currents (L-G), fault inception angle=90°

## 6. Simulation Results and Analysis

In this section, the ability of the proposed algorithm to discriminate between the inrush and fault currents is evaluated. This is simply done by testing the cases listed in the two groups. Two important cases are chosen for presenting detailed results while the remaining results are reported in table 3. These two cases are very interesting because of the transformer switching angle and the remnant flux are chosen in such away that maximum inrush current is produced. The fault inception angle is also chosen to produce maximum sub-transient current. This is clearly seen in figure 6. In this figure, the transformer is switched on at  $t = 0.05$  sec. when the remnant flux is 1.25. The transformer is subjected to a three phase fault at  $t = 0.2$  sec. For phase a, it is clear that the first peak value of the transformer inrush current is very close to that of the fault current (4 p.u.).

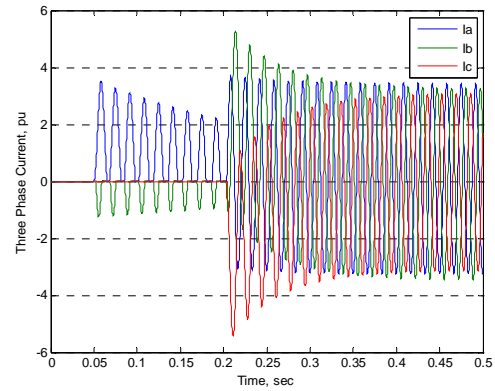


Figure 6 Three phase inrush and fault currents (3 phase fault)

Figure 7 shows the estimated p.u. fundamental component, the inrush and fault cases. Although the first peak value of both currents (phase a) are almost very close, but the fundamental components are far away from each other. It is evident that the fundamental component can be used as another indication to support the decision. Figures 8 and 9 show the variations of the per unit values of the second harmonic component for both cases. It is found that the solution reached steady state values in less than 0.003 sec. This means the acting signal, either restrain or tripping, can be issued in less than a quarter of the cycle.

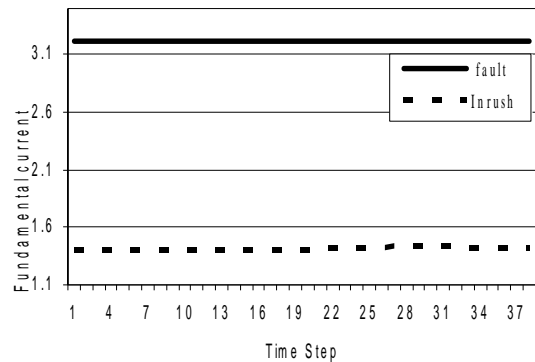


Figure 7 Fault and inrush fundamental components

Samples of the results and the critical cases are shown in table 3. In this table the actual status and the final decision of the algorithm are given. It is clear the algorithm detected all cases correctly.

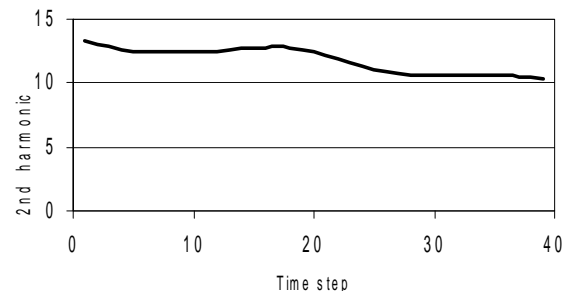


Figure 8 Inrush second harmonic components in %

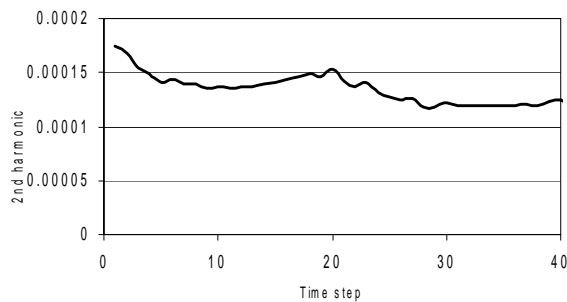


Figure 9 Second harmonic component fault current

Table 3  
Testing results

Case	Actual status	Issued Signal	Case	Actual status	Issued Signal
1	Inrush	Inrush	22	Fault	Fault
3	Inrush	Inrush	23	Fault	Fault
6	Inrush	Inrush	25	Fault	Fault
9	Inrush	Inrush	27	Fault	Fault
12	Inrush	Inrush	29	Fault	Fault
13	Inrush	Inrush	30	Fault	Fault
15	Inrush	Inrush	32	Fault	Fault
17	Fault	Fault	33	Fault	Fault
18	Fault	Fault	34	Fault	Fault
19	Fault	Fault	35	Fault	Fault
20	Fault	Fault			

## 7. Conclusion

This paper proposes a novel algorithm for digital differential protection of transformers. The algorithm is based on a dynamic digital filter that can extract signal signature in less than a quarter of a cycle. The proposed algorithm is evaluated by simulation. Different study cases are simulated. The simulation is accomplished using Simulink and SimPowerSystems block set. Results obtained show that the proposed algorithm is capable of detecting inrush currents and can differentiate between them and the fault currents with a very high degree of accuracy. The proposed algorithm has an important advantage. The advantage is the ability of the algorithm to perform the job in a very short time. Based on the power frequency (60 Hz), it is shown that the relay signal can be issued in less than a quarter of a cycle (0.0042 sec.).

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