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A NONLINEAR STEAM TURBINE MODEL FOR SIMULATION AND STATE MONITORING

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ABSTRACT
In this paper, based on the energy balance, thermodynamic principles and semi-empirical equations, nonlinear mathematical models are first developed in order to characterize the transient dynamics of steam turbines subsections. In addition, to estimate the steam thermodynamic properties such as specific enthalpy and specific entropy at different turbine stages, nonlinear functions are developed are utilized to develop the turbine model. Then, the related parameters of these functions and corresponding developed models are either determined by empirical relations or they are adjusted by applying genetic algorithms (GA) based on experimental data obtained from a complete set of field experiments. Finally, the responses of the overall turbine and generator model are compared with the response of the real plants, which validates the accuracy and performance of the proposed models over wide range of operations. The errors of the proposed functions are less than 0.1% and the overall modeling error is less than 0.3%.

KEY WORDS
Power plant; Steam turbine; Mathematical model; Genetic algorithm; Semi-empirical relations; Experimental data

1. Introduction

Evaluating the performance of a plant over different operational conditions is a key in control system design. Generally, Steam turbines, which are used in power plant applications, have a complex feature and consist of multistage steam expansion to increase the thermal efficiency. In this case, it is always difficult to predict the effects of proposed control system on the plant due to complexity of turbine structure. Therefore, developing nonlinear analytical models is necessary in order to study the turbine transient dynamics. These models can be used for control system design synthesis, performing real-time simulations and monitoring the desired states [1].

A vast collection of models is developed for long-term dynamics of steam turbines [2-4]. In many cases, the turbine models are such simplified that they only map input variables to outputs, where many intermediate variables are omitted [5]. The lack of accuracy in simplified models emerges many difficulties in control strategies and often, a satisfactory degree of precision is required to improve the overall control performance [6].

Identification techniques are widely used to develop mathematical models based on the measured data obtained from real system performance in power plant applications. System identification during normal operation without any external excitation or disruption would be an ideal target, but in many cases, using operating data for identification faces limitations and external excitation is required [7]. Assuming that parametric models are available, in this case, using soft computing methods would be helpful in order to adjust model parameters over full range of input-output operational data.

Genetic algorithms (GA) have outstanding advantages over the conventional optimization methods, which allow them to seek globally for the optimal solution. It causes that a complete system model is not required and it will be possible to find parameters of the model with nonlinearities and complicated structures [8].

In this paper, mathematical models are first developed for analysis of transient response of steam turbines subsections based on the energy balance, thermodynamic state conversion and semi-empirical equations. Then, the related parameters are either determined by empirical relations obtained from experimental data or they are adjusted by applying genetic algorithms. In addition, very simple and nonlinear functions are developed in order to estimate the steam thermodynamic properties such as specific enthalpy and specific entropy at different turbine stages, which are utilized to develop the intermediate and low pressure turbine models. The parameters of proposed functions are individually adjusted for the operation range of each subsection by using genetic algorithms as an optimization approach. Finally, the responses of the turbine and generator models are compared with the responses of the real plant in order to validate the accuracy and performance of the models over different operation conditions.

2. System Description

A steam turbine of a 440 *MW* power plant with once-through Benson type boiler is considered for the modeling approach. The steam turbine comprises high, intermediate and low-pressure sections. In addition, the system includes steam extractions, feedwater heaters, moisture separators, and the related actuators.

The high-pressure superheated steam of the turbine is responsible for energy flow and conversion results power generating in the turbine stages. The superheated steam at 535^oC and 18.6 *MPa* pressure from main steam header is the input to the high-pressure (HP) turbine. The input steam pressure drops about 0.5 *MPa* by passing through the turbine chest system.

The entered steam expands in the high-pressure turbine and is discharged into the cold reheater line. At the full load conditions, the output temperature and pressure of the high-pressure turbine is 351^oC and 5.37 *MPa*, respectively. The cold steam passes through moisture separator to become dry. The extracted moisture goes to HP heater and the cold steam for reheating is sent to reheat sections. The reheater consists of two sections and a de-superheating section is considered between them for controlling the outlet steam temperature. The reheated steam at 535^oC and with 4.83 *MPa* pressure is fed to intermediate pressure (IP) turbine. Exhaust steam from IP-turbine for the last stage expansion is fed into the low pressure (LP) turbine. The input temperature and pressure of the low pressure turbine is 289.7^oC and 0.83 *MPa*, respectively.

Extracted steam from first and second extractions of IP is sent to HP heater and de-aerator. Also, extracted steam from last IP and LP extractions are used for feedwater heating in a train of low-pressure heaters. The very low-pressure steam from the last extraction goes to main condenser to become cool and be used in generation loop again.

3. Turbine Model Development

The behavior of the subsystems can be captured in terms of the mass and energy conservation equations, semi-empirical relations and thermodynamic state conservation where a number of lumped models for each subsections of turbine represent the system dynamic.

There are many dynamic models for individual components, which are simple empirical relations between system variables with a limited number of parameters and can be validated for the steam turbine by using real system responses. In addition, an optimization approach based on genetic algorithm is performed to estimate the unknown parameters of models with more complex structure based on experimental data. With the respect to model complexity, a suitable fitness function and optimization parameters are chosen for training process, which are presented in Appendix (A).

3.1 HP Turbine Model

To develop the dynamic model of HP turbine, the pressure, mass flow rate and temperature of steam at input and output of each section is required.

The high-pressure steam enters the turbine through a stage nozzle designed to increase its velocity. The pressure drop produced at the inlet nozzle of the turbine limits the mass flow through the turbine. The relationship between mass flow and the pressure drop across the HP turbine is as follows, [9].

$$\dot{m}_{in} = \frac{K}{\sqrt{T_{in}}}\sqrt{p_{in}^2 - p_{out}^2} \quad (1)$$

Generally, Eq. (1) has a sufficient accuracy where water steam is the working fluid. The constant *K* can be obtained by the data taken from the turbine responses.

Base on experimental data, it is shown the input output pressure relation for HP turbine is a quite linear relation with the slope of 0.29475. The time constant for HP turbines are normally between 0.1 and 0.4, here the time constant is measured to be about 0.4 and therefore the transfer function of the input-output pressure is,

$$\frac{p_{out}}{p_{in}} = \frac{0.29475}{0.4s + 1} \quad (2)$$

The steam temperature at turbine output can be captured in the terms of entered steam pressure and temperature. By assuming that the steam expansion in HP-turbine is an adiabatic and isentropic process, it is simple to estimate the steam temperature at discharge of HP turbine by using ideal gas pressure-temperature relation.

$$\frac{T_{out}}{T_{in}} = \left(\frac{p_{out}}{p_{in}}\right)^{\left(\frac{k-1}{k}\right)} \quad (3)$$

The energy equation for adiabatic expansion, which relates the power output to steam energy declining by passing through the HP turbine, is as follows,

$$W_{HP} = \eta_{HP}.\dot{m}_{in}(h_{in} - h_{out}) = \eta_{HP}.C_p.\dot{m}_{in}(T_{in} - T_{out}) \quad (4)$$

If water steam is considered as an ideal gas, the generated power in HP turbine is captured as follows,

$$W_{HP} = \eta_{HP}.C_p.\dot{m}_{in}.(T_{in} + 273.15)(1 - \left(\frac{p_{out}}{p_{in}}\right)^{\left(\frac{k-1}{k}\right)}) \quad (5)$$

The nonlinear model proposed for HP turbine is a parametric model with unknown parameters, which its parameters can be adjusted by performing a training approach over a collection of input-output operational data between 154 to 440 *MW* of load. The error *E* is given by the mean value of squared difference between the target output y* and actual output y as follows,

$$E = \frac{1}{N}\sum_{j=1}^{N}(y_j^* - y_j)^2 \quad (6)$$

Where *N* is the number of entries used for training process.

The optimized value for specific heat, C_p, in order to reach the best performance at different load conditions, is obtained 2.1581 and consequently, the polytrophic expansion factor, *k*, be equal to 1.2718. The efficiency of a well-designed HP turbine is about 85 to 90 percent. A fair comparison between the experimental data and the simulation results shows that the obtained HP turbine efficiency equal to 89.31 percent is good enough to fit model responses on the real system responses. The proposed model for HP turbine is presented in Fig. (1).

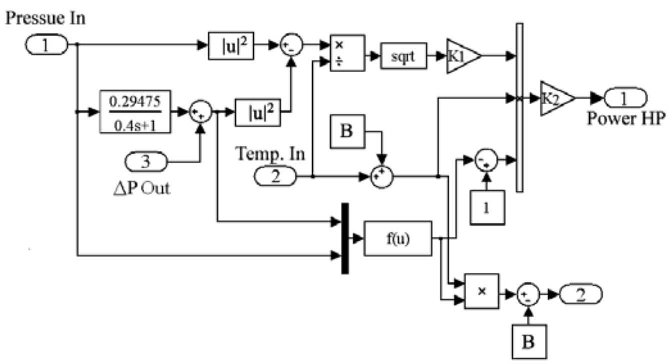

Figure 1. HP-Turbine Model
(B=273.15, K_1=520, K_2=0.1921, f(u) =$[u(1)/u(2)]^{0.2137}$)

3.2 IP and LP Turbines Model

The intermediate and low-pressure turbines have more complicated structure in where multiple extractions are employed in order to increase the thermal efficiency of turbine. The steam pressure consecutively drops across the turbine stages. The condensation effect and steam conditions at extraction stages have considerable influences on the turbine performance and generated power. In this case, developing mathematical models, which are capable to evaluate the released energy from steam expansion in turbine stages, is recommended. The steam thermodynamic properties can be estimated in term of temperature and pressure as two independent variables.

In 1988, simple formulations were presented by Garland and Hand to estimate the light water thermodynamic properties for the sub-cooled and superheated regions. In the proposed functions, saturation values of steam are used as the dominant terms in the approximation expressions [10]. The approximation functions for the thermodynamic properties in sub-cooled conditions are presented as follows,

$$F(p,T) = F_s(p_s(T)) + R(T) \times (p - p_s) \tag{7}$$

In addition, the approximation functions for the thermodynamic properties in superheated conditions are presented as follows,

$$F(p,T) = F_g(p) + R(p,T) \times (T - T_s) \tag{8}$$

The proposed functions are quite suitable for estimating the water/steam thermodynamic properties; however, these functions are tuned for a given range from 0.085 *MPa* to 21.3 *MPa* and they have not adequate accuracy for very low-pressure steam particularly for the extractions conditions.

In this paper, it is recommended that these functions be tuned individually for each input and output and at desired operational ranges. It should be mentioned that pressure changes have significant effects on the steam parameters and therefore, it is focused on adjusting the first term of functions which depend on pressure and the functions *R (T)* and *R (P, T)* are considered the same as presented by Garland and Hand which are presented in Appendix (B).

The working fluid at different turbine stages can be single or two phases. In this condition, it should be assumed that both phases of steam mixtures are in thermodynamic equilibrium and liquid and vapor phases are two separated phases. The steam conditions at each section are presented in Table (1).

Table 1 Steam Condition at Turbine Extractions

	Ext.	Pressure (Sat. Temp.)	Temperature (^oC)	Steam Condition
IP	No. 1	2.945 (233.91)	456.6	One Phase
	No. 2	1.466 (197.58)	359	One Phase
	No. 3	0.830 (171.85)	289.1	One Phase
LP	No. 4	0.301 (133.63)	182.7	One Phase
	No. 5	0.130 (105.80)	111.2	Transient
	No. 6	0.0459	77.5	Two Phases
	No. 7	0.0068	38.2	Two Phases

The proposed functions for specific enthalpy for liquid phase and specific entropy of both liquid and vapor are defined by three parameters as follows,

$$F = a(p)^b + c \tag{9}$$

In addition, the proposed function for specific enthalpy for vapor phase is defined by three parameters and one constant as follows,

$$F = a(p-d)^2 + b(p-d) + c \tag{10}$$

The parameters *a*, *b*, and *c* are adjusted for four different steam conditions at 35, 50, 75 and 100 percent of load and constant *d* is chosen manually with respect to pressure variation ranges.

The error *E* is given by the mean value of absolute difference between the target output *y** and actual output *y* as follows,

$$E = \frac{1}{N}\sum_{j=1}^{N}\left|y_j^* - y_j\right| \tag{11}$$

where *N* is the number of entries used for training process.

The proposed functions for estimating the enthalpy and entropy of the steam/water in both vapor and liquid phases are presented in Appendix (B). It should be mentioned that the errors of the proposed functions are less than 0.1 percent.

The proposed functions are depending on pressure and temperature of the steam and these variables are necessary to be defined at deferent operational conditions. In addition, the mass flow rate through the turbine stages is sequentially decreased as by subtracting extracted steam flow. The steam temperature, pressure and flow rate at extraction stages can be defined as a function of entered steam variables where first order transfer functions are accurate enough to fit the model response on the real experimental data.

For the two-phase region, the steam enthalpy of the extracted steam is depending on the steam quality. By considering expansion of steam in extraction chamber is an adiabatic process; the steam quality can be captured base on the steam entropy as follow,

$$s_0 = s_f + x.s_{fg} \quad \Rightarrow \quad x = \frac{s_0 - s_f}{s_{fg}} \qquad (12)$$

then,

$$h = h_f + x.h_{fg} \qquad (13)$$

The steam entropy at two-phase region (at fifth, sixth and seventh extractions) is considered to be equal with steam entropy at fourth extraction (one-phase region).

By considering steam expansion at turbine stages be an ideal process, the energy equations for steam expansion in turbine, which relates the power output to steam energy declining across turbine stages, can be captured. Therefore, the work done in IP turbine can be captured as follows,

$$\begin{aligned} W'_{IP} &= \dot{m}_{IP}(h_{IP} - h_{ex1}) + (\dot{m}_{IP} - \dot{m}_{ex1})(h_{ex1} - h_{ex2}) \\ &+ (\dot{m}_{IP} - \dot{m}_{ex1} - \dot{m}_{ex2})(h_{ex2} - h_{ex3}) \end{aligned} \qquad (14)$$

Now, the performance index can be considered for IP turbine.

$$W_{IP} = \eta_{IP} W'_{IP} \qquad (15)$$

The LP turbine consists of four extraction levels. The work done in the LP turbine can be captured as follows,

$$\begin{aligned} W'_{LP} &= \dot{m}_{LP}(h_{LP} - h_{ex4}) + (\dot{m}_{LP} - \dot{m}_{ex4})(h_{ex4} - h_{ex5}) \\ &+ (\dot{m}_{LP} - \dot{m}_{ex4} - \dot{m}_{ex5})(h_{ex5} - h_{ex6}) \\ &+ (\dot{m}_{LP} - \dot{m}_{ex4} - \dot{m}_{ex5} - \dot{m}_{ex6})(h_{ex6} - h_{ex7}) \end{aligned} \qquad (16)$$

where $\dot{m}_{LP} = \dot{m}_{IP} - \dot{m}_{ex1} - \dot{m}_{ex2} - \dot{m}_{ex3}$ then,

$$W_{LP} = \eta_{LP} W'_{LP} \qquad (17)$$

The optimal values for efficiencies of IP and LP turbines are obtained 83.12% and 82.84%, respectively, which are fitting turbine model responses on the real system responses.

The overall generated mechanical power can be captured by summation of generated power in turbine stages of as follows,

$$P_m = W_{HP} + W_{IP} + W_{LP} \qquad (18)$$

4. Reheater Model

Reheater section is a very large heat exchanger, which has significant thermal capacity and steam mass storage. The reheater dynamics increase nonlinearity and time delay of the turbine and should take into account as a part of turbine model.

We have developed accurate Mathematical models for subsystems of a once through Benson type boiler based on the thermodynamics principles and energy balance which are presented in [11]. The parameters of these models are determined either from constructional data such as fuel and water steam specification, or by applying genetic algorithm techniques on the experimental data. The proposed equations for the temperature model is as follow,

$$\frac{dT_{out}}{dt} = K_2(K_1\dot{m}_{fuel} + \dot{m}_{in}(T_{in} - T_{out} + B_1) + B_2) \qquad (19)$$

In this model, the steam quality has significant effects on output temperature and should be considered in related equations where the transfer function for fuel flow rate and steam quality is as follows,

$$\frac{\alpha}{\dot{m}_{fuel}} = \frac{9.45039e-6}{20s+1} \qquad (20)$$

According to the mass accumulation effects and by considering that the pressure loss due to change in flow velocity is prevailing in the steam volume, the flow-pressure model is presented as follow,

$$\frac{dp}{dt} = \frac{p_0}{\tau.m_v}(\dot{m}_{in} - \dot{m}_{out}) \qquad (21)$$

The swing of main steam flow strictly relies on the change of steam pressure as follow,

$$\frac{d\dot{m}_{out}}{dt} = \frac{\dot{m}_{out0}}{2(p_{in0} - p_{out0})}\frac{dp}{dt} \qquad (22)$$

5. Generator Model

The turbine-generator speed is described by the equation of motion of the machine rotor, which relates the system inertia to deference of the mechanical and electrical torque on the rotor.

$$P_m - P_e = M\frac{d}{dt}(\Delta\omega_m) \qquad (23)$$

It is noted that when the machine is not under active governor control, the torque-speed characteristic is nearly linear over a limited range. In this regards, no difference is declared for steady state and transient characteristic in the literature [13]. The relation between mechanical torque and mechanical power is as follows,

$$Tr_m = P_m / \omega \qquad (24)$$

In addition, it is recommended that the term of losses in rotating system be considered in the left hand side of Eq. (23) to complete the generator model. Therefore,

$$P_L = P_{L0}\left(\frac{\omega}{\omega_0}\right)^2 \qquad (25)$$

6. Simulation results

In this section, the developed model for turbine is simulated by using *Matlab Simulink*. In order to validate the accuracy and performance of the developed model, a comparison between the responses of the proposed model and the responses of the real plant is performed.

The load response in steady state and transient conditions over an operation range between 50% and 100% of nominal load is shown in Fig. (3) to illustrate the behavior of the turbine-generator system.

The simulation results indicate that the response of the developed model is very close to the response of the real system such that the maximum difference between the response of the actual system and the proposed model is much less than 0.3%.

In addition, we define the error as the difference between the response of the actual plant and the response of the model and evaluate the quantities such as norm error ($||e||$), upper bound of its absolute value ($\overline{\sigma}(e)$), lower bound of its absolute value ($\underline{\sigma}(e)$), error mean value ($M(e)$) and error covariance ($C(e)$) which are illustrated in Table (2).

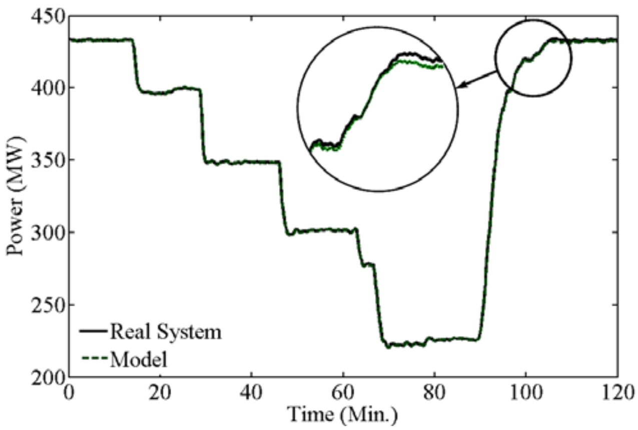

Figure 3. Response of the Turbine-Generator

Table 2 Modeling Error of Output Power

$\|\|e\|\|$	$\overline{\sigma}(e)$	$\underline{\sigma}(e)$	M (e)	C (e)
44.5054	1.6324	3.3156e-5	-0.3605	0.4556

7. Conclusion

In this paper, based on energy balance, thermodynamic state conversion and semi-empirical relations, different parametric models are developed for the steam turbine subsections. In addition, very simple and nonlinear functions are developed in order to estimate the steam thermodynamic properties such as specific enthalpy and specific entropy at different turbine stages, which are utilized to develop the turbine model. The unknown parameters of developed function and models are adjusted either bases on empirical relations or by applying genetic algorithms as optimization method.

The comparison between the responses of the turbine-generator model with the responses of the real system validates the accuracy of the proposed model in the steady state and transient conditions.

The presented turbine-generator model can be used for control system design synthesis, performing real-time simulations and monitoring the desired states in order to have safe operation of a turbine-generator particularly during the abnormal conditions such as load rejection or turbine over-speed.

The further model improvements will make the turbine-generator model proper to be used in emergency control system designing.

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Nomenclature		**Subscripts**	
C_p	Specific heat	e	Electrical
h	Specific enthalpy	ex	Extraction
J	Momentum of Inertia	f	Liquid phase
k	Index of expansion	$fuel$	Fuel
$\dot{m}$	Mass flow	g	Vapor phase
M	Inertia constant	in	Input
p	Pressure	m	Mechanical
P	Power	out	Output
Q	Heat transferred	p	Constant pressure
s	Entropy	s	Saturation
t	Time	0	Standard condition
T	Temperature	HP	High pressure
Tr	Torque	IP	Intermediate Pressure
W	Power	LP	Low Pressure
Greek letters			
α	Steam quality		
δ	Rotor angle		
η	Efficiently		
ρ	Specific density		
τ	Time constant		
ω	Frequency		

Appendix A: Optimization Parameters for GA

	HP Turbine	IP and LP Turbine	Functions
Population Size	20	50	400
Crossover Rate	0.7	0.7	0.8
Mutation Rate	0.1	0.1	0.2
Generations	50	100	2500
Selecting	Stochastic Uniform		
Reproduction	Elite Count :2		

Appendix B: Steam Thermodynamic Properties

Specific enthalpy, liquid phase:

$$h(p,T) = h_f(p_s(T)) + \left[1.4 - \frac{169}{369 - T}\right](p - p_s)$$

$h_f = 44.12782275 \times (1000p)^{0.60497549} + 19.30060027$ $\quad 0.0038\ MPa < p < 0.0068\ MPa$

$h_f = 194.57965086 \times (100p)^{0.33190979} + 1.73249442$ $\quad 0.0180\ MPa < p < 0.0459\ MPa$

$h_f = 258.51219036 \times (100p)^{0.17513608} + 11.83393526$ $\quad 0.0683\ MPa < p < 0.13\ MPa$

Specific enthalpy, vapor phase:

$$h(p,T) = h_g(p) - \left[\frac{4.5p}{\sqrt{7.4529 \times 10^{-0.6} T^3 - p^2}} + 0.28 \times e^{-0.008(T-162)} - \frac{100}{T} - 2.225\right](T - T_s)$$

$h_g = -0.48465587 \times (1000p\text{-}5)^2 + 6.47301169 \times (1000p\text{-}5) + 2560.91238452$ $\quad 0.0038\ MPa < p < 0.0068\ MPa$

$h_g = -1.82709298 \times (100p\text{-}2)^2 + 17.40365447 \times (100p\text{-}2) + 2606.680821285$ $\quad 0.0180\ MPa < p < 0.0459\ MPa$

$h_g = 0.50205745 \times (100p\text{-}12)^2 + 6.64525736 \times (100p\text{-}12) + 2679.80609322$ $\quad 0.0683\ MPa < p < 0.13\ MPa$

$h_g = -2.07829396 \times (10p\text{-}3)^2 + 25.01448122 \times (10p\text{-}3) + 2704.84920557$ $\quad 0.195\ MPa < p < 0.301\ MPa$

$h_g = -0.49047808 \times (10p\text{-}8)^2 + 10.48902998 \times (10p\text{-}8) + 2740.0576451$ $\quad 0.432\ MPa < p < 0.830\ MPa$

$h_g = -0.21681424 \times (10p\text{-}14.5)^2 + 6.13049409 \times (10p\text{-}14.5) + 2771.18901288$ $\quad 0.753\ MPa < p < 1.466\ MPa$

$h_g = -0.08217055 \times (10p\text{-}29)^2 + 3.07429644 \times (10p\text{-}29) + 2816.82024234$ $\quad 1.471\ MPa < p < 2.945\ MPa$

$h_g = -0.11673499 \times (10p\text{-}48)^2 + 0.13784178 \times (10p\text{-}48) + 2862.43339472$ $\quad 2.388\ MPa < p < 4.83\ MPa$

Specific entropy, liquid phase:

$s_f = 0.27490714 \times (1000p)^{0.60265499} - 0.22865089$ $\quad 0.0038\ MPa < p < 0.0068\ MPa$

$s_f = 1.26673390 \times (100p)^{0.17853959} - 0.61703122$ $\quad 0.0180\ MPa < p < 0.0459\ MPa$

$s_f = 0.92671704 \times (100p)^{0.14323925} - 0.03660477$ $\quad 0.0683\ MPa < p < 0.13\ MPa$

Specific entropy, vapor phase:

$$s(p,T) = s_g(p) - \left[\frac{0.004\,p^{1.2}}{\sqrt{3.025 \times 10^{1.1}(T+46)^5 - p^2}} + \frac{0.00006}{\sqrt{p}} - 4.125 \times 10^{-6} \times T + 0.0053\right](T - T_s)$$

$s_g = 8.83064734 - 0.12141594 \times (1000p)^{0.77932806}$ $\quad 0.0038\ MPa < p < 0.0068\ MPa$

$s_g = 9.0863247 - 0.96869236 \times (100p)^{0.26139247}$ $\quad 0.0180\ MPa < p < 0.0459\ MPa$

$s_g = 8.36610497 - 0.45436108 \times (100p)^{0.34246778}$ $\quad 0.0683\ MPa < p < 0.13\ MPa$

$s_g = 7.42364087 - 0.10328045 \times (10p)^{1.27827923}$ $\quad 0.195\ MPa < p < 0.301\ MPa$