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TRAJECTORY TRACKING OF NONHOLONOMIC CONSTRAINT MOBILE ROBOT BASED ON ADRC

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Abstract

The wheeled mobile robot is a typical uncertain robot system, which has typical nonholonomic characteristics and underdrive characteristics with different input and output dimensions. In the unknown environment, the slip and sideslip of the wheeled mobile robot are inevitable. Therefore, the research on trajectory tracking control of the wheeled mobile robot under sliding is of great significance. Aiming at the tracking problem of the wheeled mobile robot system with side slip disturbance, a Lyapunov design method based on extended state observer is proposed by using active disturbance rejection strategy. The influence of the sideslip disturbance on the system is regarded as the total disturbance of the system, and the extended state observer is used to observe the total disturbance in real time. The observation of the total disturbance by the extended state observer plays a patch role in the control law. When there is no disturbance, the system returns to the Lyapunov control under the nominal model. When there is a disturbance, the disturbance is compensated by the extended state observer, so that the system is approximately equivalent to the nominal model without being affected by the disturbance. The simulation results show that the system has better transient performance and anti-interference performance compared with the inverse control regardless of whether it is disturbed or not.

Key Words

ADRC; robot; trajectory tracking

1. Introduction

The wheeled mobile robot is a kind of robot that integrates sensing and control. It can complete specified tasks in a complex environment and has become one of the most widely used robots at present. The motion of the wheeled mobile robot needs to meet the pure rolling constraint between the wheel and the ground, which is a typical nonholonomic system. The wheeled mobile robot is affected by many uncertain factors in the control process. For example, there are some inherent nonlinear characteristics that are neglected in practical systems. The actual structural parameters of the system cannot be completely and accurately obtained, and there is inevitably parameter uncertainty. In addition, it is also affected by unknown external disturbances such as sideslip. Therefore, we must try to eliminate the adverse effects of system uncertainty and disturbance, which brings greater challenges to the control of the wheeled mobile robot.

Trajectory tracking of wheeled robot has always been one of the hotspots in mobile robot control research [1]–[3]. In order to achieve high-precision tracking of mobile robot, scholars have proposed many effective control methods, such as backstepping control [4], neural network control, sliding-mode control, adaptive control, etc. However, the control algorithms mentioned above are all based on the ideal condition of "pure rolling without sliding," without considering the influence of disturbance on the control performance of the mobile robot in actual operation. Therefore, when the mobile robot suffers from various serious disturbances, its control performance will inevitably be affected [5].

In order to solve the above problems, Ye *et al.* [6] proposed a cascade controller structure, which solved the trajectory tracking problem of the wheeled mobile robot under the condition of bounded external disturbance and parameter uncertainty. The robustness and effectiveness of the proposed control method are verified by simulation. Ding *et al.* [7] discussed the adaptive sliding-mode trajectory tracking control of the wheeled mobile robot with external disturbance and inertia uncertainty. Li *et al.* [8] focused on the nonlinear discrete-time mobile robot dynamic system with lateral and longitudinal sliding. An adaptive neural network [9] control tracking algorithm based on reinforcement learning is proposed. The control strategies mentioned above all improve the tracking performance of the control system to a certain extent and have good disturbance suppression ability. However, the above methods generally have the following problems: (a) There are many controller parameters and it is difficult to adjust parameters. (b) The algorithm

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is computationally intensive and needs high-performance equipment to cooperate, which increases the hardware cost.

In recent years, the observer-based disturbance rejection control method has attracted extensive attention due to its numerous advantages [10]–[12]. Commonly used observers mainly include sliding mode [13], [14] observer, disturbance observer, generalised proportional integral observer, extended state observer and so on. Wang *et al.* [15] proposed an adaptive [16], [17] tracking controller based on disturbance observer, which solved the problem of uncertain disturbance in the dynamic model of thewheeled mobile robot and effectively improved the anti-interference ability of the control system. These disturbance observers mentioned above can be used to observe the system disturbance, and the appropriate observer can be selected according to the actual situation of the studied system.

ADRC was put forward by Han Jingqing, a researcher of Chinese Academy of Sciences. Its ideology originated in 1980s [18], matured in 1990s [19], [20], and has been continuously developed in recent years. Han Jingqing put forward the controller idea of using ESO to realise internal and external disturbance compensation and introduced the concept of active disturbance rejection control. It has the advantages of simple structure, convenient parameter adjustment, and no need for accurate modelling [21]. ADRC provides a new paradigm for traditional control problems, that is, actively estimating and eliminating internal and external disturbances. With the natural advantage of anti-interference, it has been widely used in motor control [22], [23], robot arm servo control [24], [25], spacecraft flight control [26]–[28], power system control [29]–[32], missile control [33], [34], and other fields and has been highly concerned by control theorists [35]–[41]. Soudbakhsh *et al.* [42] compared the linear optimal method with the sliding-mode variable structure control method, and tracked the mobile robot at low speed and high speed. The designed sliding-mode variable structure controller can improve the control accuracy and robustness. Chen *et al.* [43] proposed an anti-saturation adaptive sliding-mode variable structure control method to eliminate the influence of parameter perturbation and uncertain factors on the control performance, so as to solve the problem of robust trajectory tracking of mobile robots, but the calculation of the adaptive law is complicated.

Aiming at the tracking problem of the wheeled mobile robot system with sideslip disturbance, a Lyapunov design method based on ESO is proposed by adopting the ADRC strategy. The influence of sideslip disturbance on the system is regarded as the total disturbance of the system, and the total disturbance is observed in real time by using the ESO. The observation of the total disturbance by the ESO plays a role of a patch in the control law, and when there is no disturbance, the system reverts to Lyapunov control under the nominal model. When there is a disturbance, the system is approximately equivalent to the nominal model without being affected by the disturbance by expanding the state observer to compensate the disturbance. Compared with the backstepping control, the simulation results show that the ADRC system has good transient performance and anti-disturbance performance whether it is disturbed or not.

The paper is organised as follows. Section myrefII shows the establishment of a mobile robot model. Section myrefIII discusses the design of trajectory tracking inversion controller for mobile robot. Section myrefIV discusses the ADRC trajectory tracking of mobile robot. It includes the design of ESO and the design of controller. A comparison of inverse control performance with active disturbance rejection control is discussed in Section myrefV. Finally, conclusions are drawn in Section myrefVI.

2. Problem Description

2.1 Two-wheel Differential Mobile Robot Model

Take the two-wheeled differential mobile robot as the research object, considering only the movement on the two-dimensional horizontal plane, and its kinematic model is shown in Fig. 1. XOY is a generalised coordinate system, 2b is the width of the vehicle body, and r is the wheel radius. The attitude of the mobile robot can be described as $q = [x\ y\ \varphi\ \theta_r\ \theta_l]^T$. In which (x, y) is the coordinate of P point at the midpoint of left and right driving wheels, Φ is the heading angle, θ_r is the right wheel rotation angle, θ_l is the left wheel rotation angle, F is the linear velocity direction of the mobile robot, and it is always perpendicular to the axle connecting line. The robot is controlled by two independently driven rear wheels, and the input is linear velocity v and angular velocity ω. The pose of the mobile robot can be changed by controlling the input.

Assuming that there is only rolling and no sliding when the mobile robot moves, three constraints are met, and the linear velocity direction is always on the central axis of the mobile robot. The constraints are shown in the following equation:

$$\begin{aligned} \dot{y}\cos\phi - \dot{x}\sin\phi &= 0 \\ \dot{x}\cos\phi + \dot{y}\sin\phi + b\dot{\varphi} &= r\dot{\theta}_1, \\ \dot{x}\cos\phi + \dot{y}\sin\phi - b\dot{\varphi} &= r\dot{\theta}_1 \end{aligned} \tag{1}$$

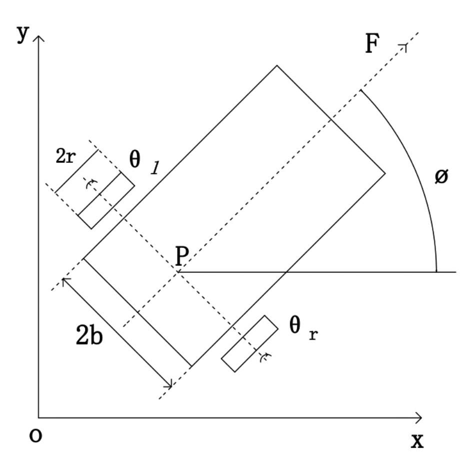

Figure 1. Two-wheel differential mobile robot model.

Constraints can be expressed as:

$$A(q)\dot{q} = 0, \tag{2}$$

where:

$$A(q) = \begin{bmatrix} \sin\phi & -\cos\phi & 0 & 0 & 0 \\ \cos\phi & \sin\phi & b & -r & 0 \\ \cos\phi & \sin\phi & -b & 0 & -r \end{bmatrix} \tag{3}$$

Therefore, the kinematics model of nonholonomic-constrained robot can be expressed as (4) and (5):

$$\dot{q} = S(q)\nu(t) \tag{4}$$

$$M(q)\dot{\nu} + V(q,\dot{q})\nu = B(q)\tau \tag{5}$$

where $S(q) \in R^{n\times(n-m)}$ is a full-rank matrix composed of a set of linearly independent vectors, and the expression is (6). $\nu(t)$ is the velocity vector. $q \in R^n$ is the generalised coordinate, $\tau \in R^\lambda$ is the input vector, where $\lambda = n - m, M(q) \in R^{n\times n}$ are the symmetric positive-definite inertia matrices, $V(q,\dot{q}) \in R^{n\times n}$ is the centripetal force and Coriolis force matrices, and $B(q) \in R^{n\times n}$ is the input transformation matrix.

$$S(q) = \begin{bmatrix} \frac{r}{2}\cos\phi & \frac{r}{2}\cos\phi \\ \frac{r}{2}\sin\phi & \frac{r}{2}\sin\phi \\ \frac{r}{2b} & -\frac{r}{2b} \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{6}$$

2.2 Kinematics Model of Mobile Robot

With v as the control input, the motion trajectory is constructed as the following equation:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \\ \dot{\theta}_l \\ \dot{\theta}_r \end{bmatrix} = \begin{bmatrix} \frac{r}{2}\cos\phi & \frac{r}{2}\cos\phi \\ \frac{r}{2}\sin\phi & \frac{r}{2}\sin\phi \\ \frac{r}{2b} & -\frac{r}{2b} \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix} \tag{7}$$

where v_1 and v_2 are the angular velocities of the left wheel and the right wheel of the mobile robot respectively.

If only x, y, and φ are considered, θ_l and θ_r are ignored. The relationship between v_1, v_2 and the linear velocity v, angular velocity ω of the mobile robot can be expressed as (8):

$$\begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{r} & \frac{b}{r} \\ \frac{1}{r} & -\frac{b}{r} \end{bmatrix} \begin{bmatrix} \nu \\ \omega \end{bmatrix} \tag{8}$$

Substituting (8) into (7), the general form of the kinematic model of the mobile robot is shown in (9).

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \cos\phi & 0 \\ \sin\phi & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \nu \\ \omega \end{bmatrix} \tag{9}$$

3. Units Inversion Tracking Control

3.1 Design of Backstepping Controller Under Sideslip Condition

Considering the kinematic model shown in (9), the given trajectory is:

$$\begin{cases} x_d = \sin\phi_d \\ y_d = -\cos\phi_d \end{cases} \tag{10}$$

where x_d and y_d represent the ideal angles in the X-axis and the Y-axis direction, and φ_d is an ideal pose angle. Coordinate (x_d, y_d) and angle φ_d are not independent of each other, but two of the three variables are independent. The selected position instruction is (x_d, y_d) and the position tracking error is (x_e, y_e).

The inversion controller is designed as follows:

1. Introduce the virtual input α, (10) is obtained from (9).

$$\begin{cases} \dot{x} = v\cos\alpha \\ \dot{y} = v\sin\alpha \end{cases} \tag{11}$$

let the Lyapunov function be:

$$V_1 = \frac{1}{2}x_c^2 + \frac{1}{2}y_c^2 \tag{12}$$

where $x_e = x_d - x, y_e = y_d - y$.

Equation (13) can be obtained from (11) and (12):

$$\begin{aligned} \dot{V}_1 &= x_e\dot{x}_e + y_e\dot{y}_e \\ &= x_e(\dot{x}_d - v\cos\alpha) + y_e(\dot{y}_d - v\sin\alpha) \end{aligned} \tag{13}$$

by designing the virtual quantity α, we can make the following equation:

$$\begin{aligned} v\cos\alpha &= \dot{x}_d + c_1x_e \\ v\sin\alpha &= \dot{y}_d + c_2y_e \end{aligned} \tag{14}$$

Then

$$\dot{V}_1 = -c_1x_e^2 - c_2y_e^2 < 0 \tag{15}$$

Let $m_1 = \dot{x}_d + c_1x_e, m_2 = \dot{y}_d + c_2y_e$, if the linear velocity and virtual control rate are designed as (16) and (17):

$$v = \sqrt{m_1^2 + m_2^2} \tag{16}$$

$$\alpha = \arctan\frac{m_2}{m_1} = \arctan\frac{\dot{y}_d + c_2y_e}{\dot{x}_d + c_1x_e} \tag{17}$$

Then, (14) can be guaranteed.

It can be seen that if $x_e = 0, y_e = 0$, then $\alpha = \arctan \frac{\dot{y}_d}{\dot{x}_d} = \phi_d$, in order to realise φ tracking φ_d, the second step is to ensure φ tracking α.

2. Let $e = \alpha - \phi$, and define the Lyapunov function as:

$$V_2 = V_1 + \frac{1}{2}e^2 \tag{18}$$

Then

$$\dot{V}_2 = -c_1 x_e^2 - c_2 y_e^2 + e(\dot{\alpha} - w) \tag{19}$$

The angular velocity control law is designed as

$$w = \dot{\alpha} + c_3 e \tag{20}$$

Then

$$\dot{V}_2 = -c_1 x_e^2 - c y_c^2 - c_3 e^2 \leqslant -2C_m V_z$$

where $C_m \leqslant min\,(C_1, C_2, C_3)$.

Then $V_2(t) \leqslant e^{-2C_m t} V_2(0)$ converges to zero exponentially, so when $t \to \infty, x_e \to 0, y_e \to 0, \phi \to \phi_d$ and converges exponentially.

4. ADRC Tracking Control LADRC

4.1 Tracking Control Under Sideslip Condition

When there is sideslip, the kinematic model equation (9) of the mobile robot becomes (21):

$$\begin{cases} \dot{x}_s = \nu \cos(\phi + \varepsilon_1) \\ \dot{y}_s = \nu \sin(\phi + \varepsilon_1) \\ \dot{\phi} = \omega + \varepsilon_2 \end{cases} \tag{21}$$

where ε_1 is the slip angle and ε_2 is the disturbance yaw rate. The control goal is to design control laws v and ω to make the mobile robot system track the desired trajectory. The kinematic equation of the desired trajectory is as:

$$\begin{cases} \dot{x}_d = \nu_d \cos\phi_d \\ \dot{y}_d = \nu_d \sin\phi_d \\ \dot{\phi} = \omega_d \end{cases} \tag{22}$$

The tracking error is defined as:

$$\begin{bmatrix} \widetilde{x} \\ \widetilde{y} \\ \widetilde{\phi} \end{bmatrix} = \begin{bmatrix} \cos(\phi + \varepsilon_1) & \sin(\phi + \varepsilon_1) & 0 \\ -\sin(\phi + \varepsilon_1) & \cos(\phi + \varepsilon_1) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_d - x \\ y_d - y \\ \phi_d - (\phi + \varepsilon_1) \end{bmatrix} \tag{23}$$

where $q_d = \begin{bmatrix} x_d \ y_d \ \phi_d \end{bmatrix}^T$ is the desired trajectory coordinate.

Derive the tracking error formula (23) and get (24)–(26):

$$\begin{aligned} \dot{\widetilde{x}} &= \cos(\phi + \varepsilon_1)(\dot{x}_d - \dot{x}) + \sin(\phi + \varepsilon_1)(\dot{y}_d - \dot{y}) \\ &\quad - \sin(\phi + \varepsilon_1)\left(\dot{\phi} + \dot{\varepsilon}_1\right)(x_d - x) \\ &\quad + \cos(\phi + \varepsilon_1)\left(\dot{\phi} + \dot{\varepsilon}_1\right)(y_d - y) \\ &= \dot{x}_d \cos(\phi + \varepsilon_1) - \dot{x}\cos(\phi + \varepsilon_1) + \dot{y}_d \sin(\phi + \varepsilon_1) \\ &\quad - \dot{y}\sin(\phi + \varepsilon_1) + \widetilde{y}(\omega + \varepsilon_2 + \varepsilon_1) \\ &= \widetilde{y}(\omega + \varepsilon_2 + \dot{\varepsilon}_1) + v_d \cos(\widetilde{\phi}) - v \end{aligned} \tag{24}$$

$$\begin{aligned} \dot{\widetilde{y}} &= -\sin(\phi + \varepsilon_1) = (\dot{x}_d + \dot{x}) + \cos(\phi + \varepsilon_1)(\dot{y}_d - \dot{y}) \\ &\quad - \cos(\phi + \varepsilon_1)\left(\dot{\phi} + \dot{\varepsilon}_1\right)(x_d - x) \\ &\quad - \sin(\phi + \varepsilon_1)\left(\dot{\phi} + \dot{\varepsilon}_1\right)(y_d - y) \\ &= -\dot{x}_d \sin(\phi + \varepsilon_1) + \dot{x}\sin(\phi + \varepsilon_1) + \dot{y}_d \cos(\phi + \varepsilon_1) \\ &\quad - \dot{y}\cos(\phi + \varepsilon_1) - \widetilde{x}(\omega + \varepsilon_2 + \dot{\varepsilon}_1) \\ &= -\widetilde{x}(\omega + \varepsilon_2 + \dot{\varepsilon}_1) + \nu_d \sin(\widetilde{\phi}) \end{aligned} \tag{25}$$

$$\dot{\widetilde{\phi}} = \omega_d - \omega - \varepsilon_2 - \dot{\varepsilon}_1 \tag{26}$$

Therefore, when there is sideslip, the tracking error system equation is:

$$\begin{cases} \dot{\widetilde{x}} = \widetilde{y}(\omega + \varepsilon_2 + \dot{\varepsilon}_1) + v_d \cos(\widetilde{\phi}) - v \\ \dot{\widetilde{y}} = -\widetilde{x}(\omega + \varepsilon_2 + \dot{\varepsilon}_1) + v_d \sin(\dot{\phi}) \\ \dot{\widetilde{\phi}} = \omega_d - \omega - (\varepsilon_2 + \dot{\varepsilon}_1) \end{cases} \tag{27}$$

Equation (27) describes the differential equation of tracking error of the wheeled mobile robot in the case of sideslip. The control task is to design the speed controllers v and ω to make the mobile robot track the desired trajectory, that is, the tracking error gradually approaches zero.

4.2 Design of ADRC Based on ESO

It can be seen from (27) that under the influence of sideslip, the tracking error system equation of the mobile robot is affected by the disturbance term, and the tracking problem is transformed into the stabilisation problem of (27). However, due to the existence of disturbance, it brings some difficulties to the controller design. Firstly, the extended state observer is used to observe the total disturbance of the system, and the Lyapunov controller is further designed and disturbance compensation is made in the control law, which can make the error system asymptotically stable.

The disturbance $-(\varepsilon_2 + \dot{\varepsilon}_1)$ is regarded as the total disturbance ε, so the tracking error system equation is:

$$\begin{cases} \dot{\widetilde{x}} = -\widetilde{y}(\omega - \varepsilon) + v_d \cos(\widetilde{\phi}) - v \\ \dot{\widetilde{y}} = -\widetilde{x}(\omega - \varepsilon) + v_d \sin(\widetilde{\phi}) \\ \dot{\widetilde{\phi}} = \omega_d - \omega + \varepsilon \end{cases} \tag{28}$$

The ESO design equation is:

$$\begin{cases} \dot{z}_1 = \omega_d - \omega + \varepsilon_2 - \beta_1 \left(z_1 - \widetilde{\phi} \right) \\ \dot{z}_2 = -\beta_2 \left(z_1 - \widetilde{\phi} \right) \end{cases} \tag{29}$$

where z_1 and z_2 are the estimates of $\widetilde{\phi}$ and ε, respectively, β_1 and β_2 are the gains of the observer, respectively.

The controller is designed by the Lyapunov direct method. Select the Lyapunov function as:

$$V = \frac{1}{2}\widetilde{x}^2 + \frac{1}{2}\widetilde{y}^2 + \frac{1}{k_2}(1 - \cos\widetilde{\phi}) \tag{30}$$

Derive the Lyapunov function along the tracking error equation, and we can get:

$$\begin{aligned} V &= \widetilde{x}\dot{\widetilde{x}} + \widetilde{y}\dot{\widetilde{y}} + \tfrac{1}{k_2}\dot{\widetilde{\phi}}\sin\widetilde{\phi} \\ &= \widetilde{x}\left(\widetilde{y}\left(\omega + \varepsilon_2 + \dot{\varepsilon}_1\right) - v + v_d\cos\widetilde{\phi}\right) \\ &\quad + \widetilde{y}\left(-\widetilde{x}\left(\omega + \varepsilon_2 + \dot{\varepsilon}_1\right) + v_d\sin\widetilde{\phi}\right) \\ &\quad + \tfrac{1}{k_2}\sin\widetilde{\phi}\left(\omega_d - \omega - \varepsilon_2 - \dot{\varepsilon}_1\right) \\ &= \widetilde{x}\left(-v + v_d\cos\widetilde{\phi}\right) + \widetilde{y}\left(v_d\sin\widetilde{\phi}\right) \\ &\quad + \tfrac{1}{k_2}\sin\widetilde{\phi}\left(\omega_d - \omega - \varepsilon_2 - \dot{\varepsilon}_1\right) \\ &= \widetilde{x}\left(-v + v_d\cos\widetilde{\phi}\right) + \widetilde{y}\left(v_d\sin\widetilde{\phi}\right) \\ &\quad + \tfrac{1}{k_2}\sin\widetilde{\phi}\left(\omega_d - \omega + \varepsilon\right) \end{aligned} \tag{31}$$

According to (31), the controller can be designed as (32):

$$\begin{cases} v = v_d\cos\widetilde{\phi} + k_1\widetilde{x} \\ \omega = \omega_d + k_2 v_d\widetilde{y} + k_3\sin\widetilde{\phi} + z_2 \end{cases} \tag{32}$$

where k_1, k_2, and k_3 are controller gains. z_2 is the compensation for the total disturbance. Through the above controller, the system can be gradually stabilised, and the tracking error gradually converges to zero.

5. Simulation and Analysis

5.1 Inversion Tracking Simulation Under Sideslip

If $x_d = \sin(t)$ and $y_d = -\cos(t)$, then $\alpha = \arctan\frac{\dot{y}_d}{\dot{x}_d} = \phi_d$ and the given trajectory is a circle with radius 1. The control law is given by (16) and (20), and the control parameters $c_1 = c_2 = 10$ and $c_3 = 50$. The initial value of the mobile robot is $x(0) = 0, y(0) = 0 - 1, \omega(0) = 0$.

Add sideslip disturbance at 5 s, and the specific expression is:

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} = \begin{cases} \begin{bmatrix} 0 \\ 0 \end{bmatrix}, t < 5 \\ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, t > 5 \end{cases} \tag{33}$$

The simulation results are shown in Figs. 2–6. The dotted line in Figs. 2 and 3 is the set value curve, and the solid line is the tracking curve. The midpoint in Fig. 4 is the set value curve, and the triangle is the tracking curve.

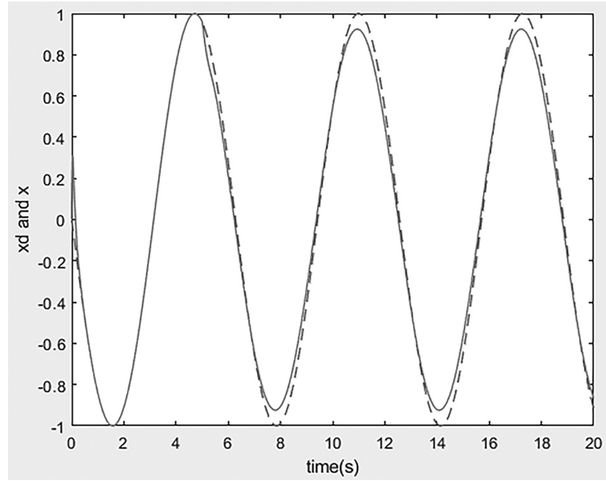

Figure 2. X-axis position tracking curve.

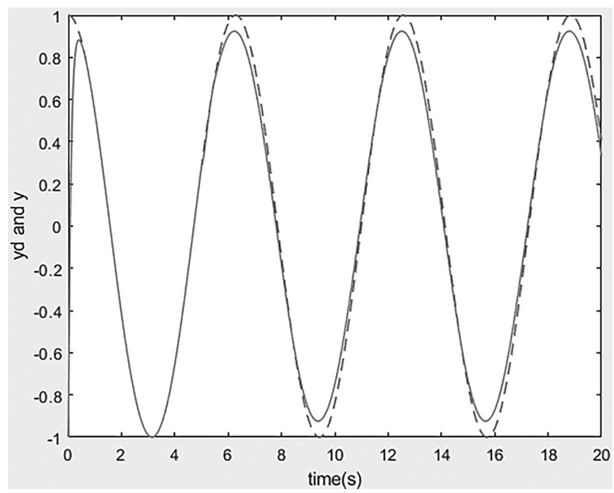

Figure 3. Y-axis position tracking curve.

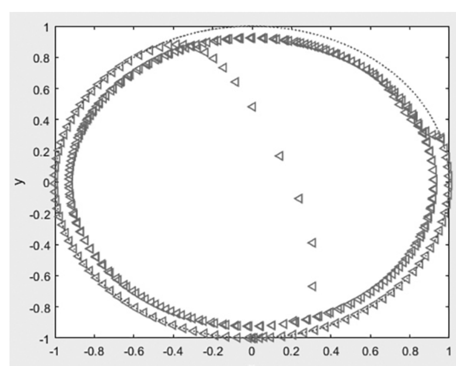

Figure 4. Position trajectory tracking curve.

5.2 ADRC Tracking Simulation in Sideslip

In order to verify the proposed Lyapunov method compound control algorithm combined with the extended

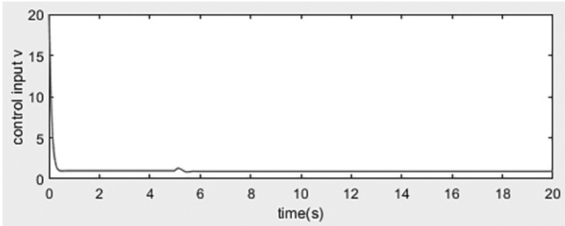

Figure 5. Control input v.

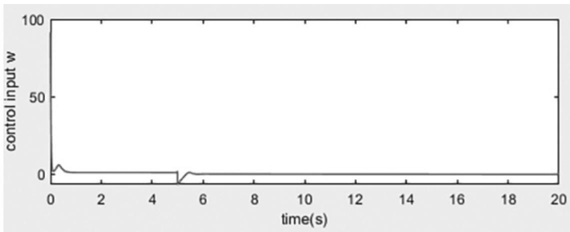

Figure 6. Control input ω.

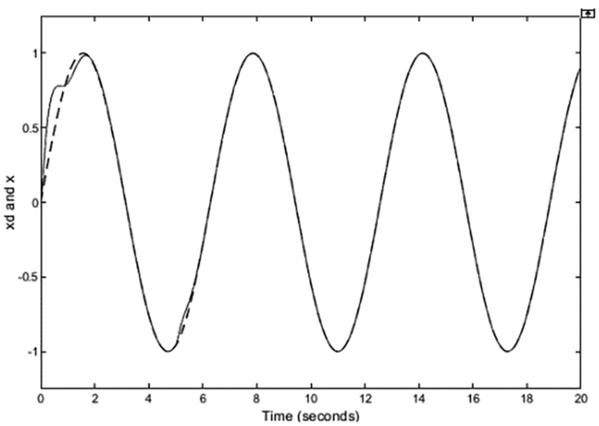

Figure 7. X-axis position tracking curve.

state observer, the circular trajectory is tracked for simulation, and the sideslip disturbance is added in the process. The circular trajectory equation tracked by the mobile robot is as follows.

When $t = 5$ s, the disturbance with amplitude of 1 is added, and the initial position of the mobile robot is $x(0) = 0, y(0) = 0 - 1, \omega(0) = 0$.

The extended state observer of (29) and the compound controller of (32) are used for simulation. The observer parameter selection is $\beta_1 = 20$, $\beta_2 = 100$, and the controller parameter selection is $k_1 = 10$, $k_2 = 5$, $k_3 = 4$.

The simulation results are shown in Figs. 7–11. The dotted line in Figs. 7 and 8 is the set value curve, and the solid line is the tracking curve.

5.3 Simulation Analysis of Sideslip

From Figs. 2 and 3, Figs. 7 and 8, it can be seen that in the tracking control on the X-axis and Y-axis, the inversion control has good tracking characteristics before sideslip interference. However, when $t = 5$ s, the tracking

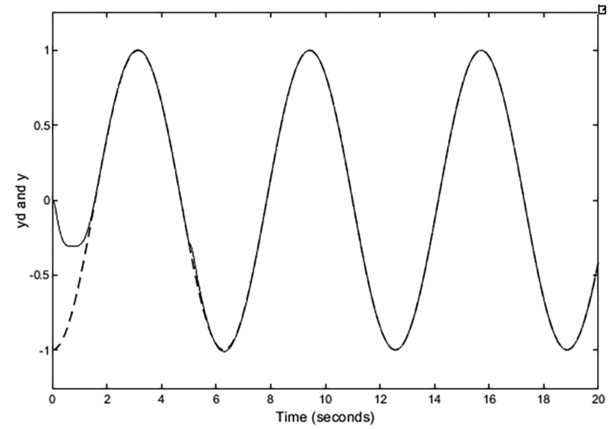

Figure 8. Y-axis position tracking curve.

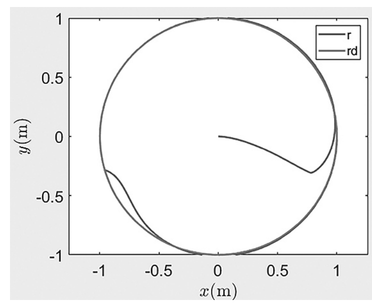

Figure 9. Position trajectory tracking curve.

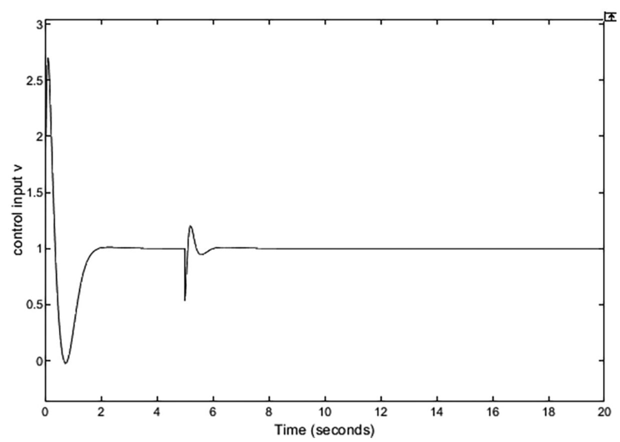

Figure 10. Control input v.

error of backstepping control appears when the sideslip interference with amplitude 1 is added, which is smaller than the expected value in both the X-axis and the Y-axis. However, ADRC adds sideslip interference when $t = 5$ s, which makes the mobile robot slightly deviate from the expected values of the X-axis and Y-axis, but it can recover to the expected values of the X-axis and Y-axis in a short time. The comparison table of the maximum tracking errors of the two control algorithms after stabilisation is shown in Table 1.

As can be seen from Figs. 4 and 9, when the mobile robot tracks the circular reference trajectory, the inversion control can track the expected trajectory well before adding

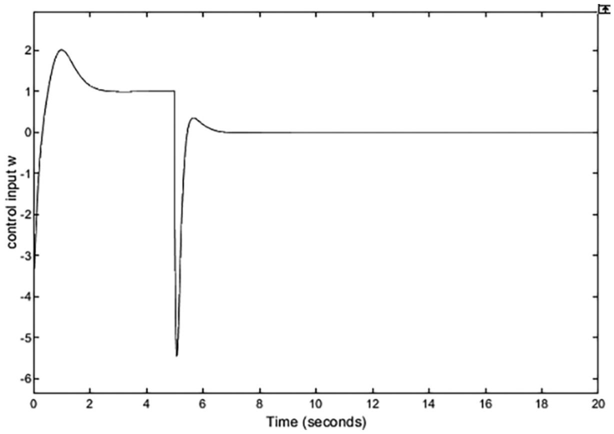

Figure 11. Control input ω.

Table 1
Comparison of Maximum Tracking Error of Two Control Algorithms After Stabilisation

Parameter	**Backstepping Controller**	**ADRC**
X-axis position tracking	0.3	0
Y-axis position tracking	0.3	0

Table 2
Controller Outputs of Two Control Algorithms

Parameter		**Backstepping Controller**	**ADRC**
$t = 0$ s	v	20	2.7
	ω	1.4	1.3
$t = 5$ s	v	87	2
	ω	0	-5.5

the sideslip interference. However, at $t = 5$ s, the sideslip interference is added, and the inversion control is smaller than the expected value in both the X-axis and the Y-axis, so that the actual trajectory is smaller than the expected trajectory and becomes a shrinking circle. However, ADRC adds sideslip interference when $t = 5$ s, which makes the mobile robot slightly deviate from the reference trajectory. But it can return to the reference trajectory in a short time.

From Figs. 5 and 6 and Figs. 10 and 11, it can be seen that the control input v and ω of the system, the inversion control needs to input a larger control quantity when tracking the initial value, so as to track the expected value. When the sideslip tracking is disturbed, the tracking error still exists when the input control quantity is larger. ADRC can track the expected value effectively, and the input control quantity is small when tracking the initial value and tracking the sideslip disturbance. The comparison of controller outputs of the two control algorithms is shown in Table 2.

In a word, the proposed Lyapunov compound controller based on the extended state observer has good anti-disturbance performance. The total disturbance of the system is observed by the extended state observer and compensated in the controller, which can make the system have good anti-disturbance performance. In addition, the controller designed by the Lyapunov direct method can make the system asymptotically stable, and the extended state observer plays a patch role in the compensation of disturbance. Without disturbance, the system has good dynamic performance. When affected by disturbance, the dynamic performance of the system is almost unaffected due to the compensation of the extended state observer for disturbance.

6. Conclusion

As a typical nonholonomic dynamic system, the nonholonomic constraints faced by wheeled mobile robots, as well as the uncertainties of parameters and sideslip disturbances, bring challenges to the controller design. Aiming at the tracking problem of the wheeled mobile robot system with sideslip disturbance, a Lyapunov design method based on the extended state observer is proposed by using the active disturbance rejection strategy. An extended state observer is designed to observe the total disturbance caused by the sideslip. Then the disturbance compensation is implemented in the Lyapunov controller so that the system is asymptotically stable while suppressing the sideslip disturbance and improving the anti-jamming performance of the system. Compared with inverse control, the system has better transient performance and anti-interference performance regardless of whether it is disturbed or not. It provides reference and reference for ADRC solutions of multi-source uncertainty problems, such as parameter uncertainty, unmodeled dynamics, and unknown external disturbances of robot systems.

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