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# AN ENHANCED PARTICLE SWARM OPTIMIZATION FOR DYNAMIC ECONOMIC DISPATCH PROBLEM CONSIDERING VALVE-POINT LOADING

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**ABSTRACT**
This paper proposes an enhanced Particle Swarm Optimization (EPSO), where a modified heuristic search method is incorporated with Particle Swarm Optimization (PSO) in order to address the Dynamic Economic Dispatch (DED) problem, while it is also aimed at overcoming the deficiency of the solution feasibility. To verify the performance of the proposed EPSO, it is tested on the 10-unit systems considering both smooth and non-smooth cost functions characteristic. The results from optimizing the standard test systems show that the proposed technique is indeed better than other approaches in terms of the solution quality. Thus, it can be concluded that the EPSO is recommended to be a promising method for solving the DED problem.

**KEY WORDS**
Particle Swarm Optimization (PSO), Dynamic Economic Dispatch (DED)

## Nomenclature

| | |
|---|---|
| $TC$ | total production cost , |
| $F_{it}(P_{it})$ | fuel cost of $i^{th}$ generator at hour $t$, |
| $P_{it}$ | power output of $i^{th}$ generator at hour $t$, |
| $P_{Dt}$ | power demand at hour $t$, |
| $P_{i(\min)}$ | minimum power output of $i^{th}$ generator, |
| $P_{i(\max)}$ | maximum power out put of $i^{th}$ generator, |
| $UR_i$ | upper limits of ramp rate of $i^{th}$ generator, |
| $DR_i$ | lower limits of ramp rate of $i^{th}$ generator, |
| $N$ | number of generators, |
| $T$ | number of hours, |
| $v_{id}^t$ | velocity of $i^{th}$ particle at iteration $t$ in $d$-dimensional space, |
| $x_{id}^t$ | current position of $i^{th}$ particle at iteration $t$ in $d$-dimensional space, |
| $w$ | inertia weight factor, |
| $t$ | number of iterations, |
| $k$ | constriction factor, |
| $c_1, c_2$ | acceleration constant. |

## 1. Introduction

Today the modern power system is more dynamic and its operation is a subject to a number of constraints that are reflected in various management and planning tools used by system operators. In the case of hourly generation planning, the traditional Static Economic Dispatch (SED) allocates the outputs of all committed generating units by considering the static behaviour of them, while the Dynamic Economic Dispatch (DED) schedules the generating outputs of all on-line units over a time horizon by taking the ramp rate constraints into account. That makes the DED problem more difficult [1-3]. Thus, the accurate solutions of the DED are essential in order to operate the power system in an economic and efficient manner.

Up to now, a number of computation techniques have progressively been proposed to solve this critical issue, for example Linear Programming [4], Lagrangian Relaxation [5], Genetic Algorithm (GA) [6], Simulated Annealing (SA) [7], Evolutionary Programming (EP) [2], etc. One of them is a Particle Swarm Optimization (PSO), which belongs to the evolutionary computation techniques, and it has attracted a great attention of the research community since it has been found to be extremely effective in solving a wide range of engineering problems. The attractive characteristics of PSO include: ease of implementation, fast convergence compared with the traditional evolutionary computation techniques and stable convergence characteristic.

This paper proposes the application of PSO to the DED problem, which occurs in the operational planning of power systems. To solve the DED problem, the PSO algorithm is firstly adopted. Subsequently, a modified heuristic search is utilized to enhance the PSO performance by dealing with the operating constraints including the ramp rate constraints. The strength of the modified version is that it increases a possibility of generating feasible solutions. To investigate the efficiency of the proposed method, the standard test systems with smooth and non-smooth cost functions have been considered.

The organization of this paper is as follows: section 2 presents the problem formulation of DED problem, while in section 3 briefly presents the details of the different cost function characteristics. Section 4 the overview of PSO algorithm is introduced. Section 5, the implementation of the proposed EPSO algorithm is presented. Simulation results will be shown and discussed in section 6. Finally, a summary is made in section 7.

## 2. Problem Formulation

Dynamic Economic Dispatch (DED) problem is to determine the optimum scheduling of generation at a certain period of time that minimizes the total production cost while satisfying equality and inequality constraints, i.e. power balance and operating limits, and ramp rate constraints, respectively. In general, the mathematical model of the DED problem is as follows [2]:

$$Minimize : TC = \sum_{t=1}^{T}\sum_{i=1}^{N} F_{it}(P_{it}) \tag{1}$$

Subject to:

a) Power balance constraint

$$\sum_{i=1}^{N} P_{it} = P_{Dt} \tag{2}$$

b) Operating limit constraints

$$P_{i(\min)} \le P_{it} \le P_{i(\max)} \tag{3}$$

c) Ramp rate constraints

$$-DR_i \le P_{i,t} - P_{i,t-1} \le UR_i \tag{4}$$

## 3. Characteristics of Cost Function

From the different characteristics of cost function; therefore, they can be categorized as DED problem with smooth cost functions (the standard DED) and DED problem with non-smooth cost functions (the practical DED) as presented in [1-4].

### 3.1 Smooth cost function

For the sake of simplicity, the cost function of the standard DED problem (smooth cost functions) is generally a single quadratic function. The generator's fuel cost function can be represented by [1, 4, 8]:

$$F_i(P_i) = a_i P_i^2 + b_i P_i + c_i. \tag{5}$$

### 3.2 Non-smooth cost function

In some large generators, their cost functions are also non-linear, due to the effect of valve-point loading [9]. Taking the valve point loading into account will increase multiple local minimum points in the cost function and make the problem more difficult [10]. The fuel cost function with valve-point loading can be expressed as [2, 3, 11]:

$$F_i(P_i) = a_i P_i^2 + b_i P_i + c_i + \left| e_i \times \sin(f_i \times (P_{i(\min)} - P_i)) \right|. \tag{6}$$

The example of both smooth and non-smooth cost functions is shown in Fig. 1.

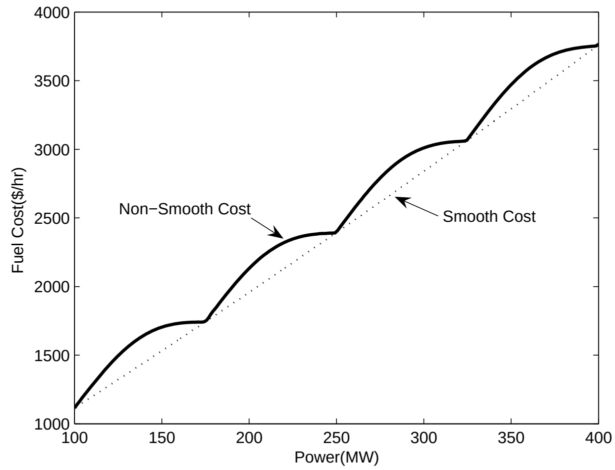


Figure 1. The plotting of cost curve with smooth and non-smooth cost characteristics

## 4. Particle Swarm Optimization

Particle Swarm Optimization (PSO) [12] is one of the modern algorithms used to solve global optimization problems [13], and it is based on similar principles as the previous methods. Thus, to solve an optimization problem, PSO applies a simplified social model, which for instance Zoologists might use to explain the movement of individuals within a group [14]. To begin with, PSO initializes a population of random solutions each of which is defined as a "particle". Initially, every particle flies into a problem hyperspace at a random velocity. Thereafter, each particle adjusts its travelling speed dynamically corresponding to the flying experiences of itself and its colleagues [15, 16]. The PSO computation will keep updating the velocity and position of the particles until it finds a global optimal solution. Therefore, the updated velocity and position of each particle can be calculated as following equations [13, 17-19]:

$$\begin{aligned} v_{id}^{t+1} = k \times [w \cdot v_{id}^{t} + c_1 \times rand_1 \times (pbest_{id} - x_{id}^{t}) \\ + c_2 \times rand_2 \times (gbest_d - x_{id}^{t})], \end{aligned} \tag{7}$$

$$x_{id}^{t+1} = x_{id}^{t} + v_{id}^{t+1}. \tag{8}$$

Constriction factor ($k$) is expressed by:

$$k = \frac{2}{\left| 2 - \varphi - \sqrt{\varphi^2 - 4\varphi} \right|},\ \varphi = c_1 + c_2,\ \varphi > 4, \tag{9}$$

where $\varphi$ is generally set to 4.1, both $c_1$ and $c_2$ are set to 2.05 and $k$ is 0.729 as presented in [20].

## 5. Implementation of EPSO algorithm in DED problem

The EPSO technique utilized an idea of the incorporation of a modified heuristic method into the basic PSO algorithm so as to address the DED problem. In this research, the standard PSO has been adopted and also incorporated with a modified heuristic search, which is modified and developed from [14] and [21], for manipulating the equality and inequality constraints. The strength of the modified version is that it increases a possibility of generating feasible solutions by employing the principle of randomization. The implementation of the proposed EPSO method can therefore be expressed in details as follows:

### 5.1 Step 1. Initialization

Step 1.1: Initialize the system data and parameters of the PSO algorithm (e.g. population size (*Pop*), initial / final inertia weight ($w_{\max}$, $w_{\min}$), acceleration constant ($c_1$ and $c_2$), and constriction factor ($k$),

Step 1.2: Randomly initialize positions ($P_{ij}$) and velocities ($v_{ij}$) of each particle in $i^{th}$ hour of $j^{th}$ unit,

Step 1.3: Define each particle as *pbest*, and the best position of all particles as *gbest*.

### 5.2 Step 2.

Update the velocity and the position for each particle using (7) and (8).

### 5.3 Step 3. Modify the positions of the particle

Step 3.1: Set $i = 1$ and $j = 1$, where $i = 1, 2, ..., T$ and $j = 1, 2, ..., N$,

Step 3.2: Randomly select $L$-th generator,

Step 3.3: Calculate $P_{iL}$ using $P_{iL} = P_D - \sum_{\substack{j=1 \\ j \neq L}}^{N} P_{ij}$,

Step 3.4: Adapt $P_{iL}$ for its operating limit if $P_{iL} < P_{iL(\min)}$ or $P_{iL} > P_{iL(\max)}$. Otherwise, go to Step 3.8,

Step 3.5: If $j \leq$ *total number of generators* ($N$), let $j = j+1$. Otherwise go to Step 3.8,

Step 3.6: Re-random $L$-th generator and re-calculate $P_{iL}$,

Step 3.7: Adjust the value of $P_{iL}$ if it is out of operating limit, and then return to the Step 3.5. Otherwise, go to the next step,

Step 3.8: Calculate the operating limit for the next hour considering ramp rate constraints from $P_{i+1,j(\min)} = P_{ij} - DR_i$ and $P_{i+1,j(\max)} = P_{ij} + UR_i$,

Step 3.9: If $P_{i+1,j(\min)} < P_{j(\min)}$, then let $P_{i+1,j(\min)} = P_{j(\min)}$ or $P_{i+1,j(\max)} > P_{j(\max)}$ then let $P_{i+1,j(\max)} = P_{j(\max)}$,

Step 3.10: If $i$ = *total number of hours* ($T$), then go to Step 4. Otherwise, let $i = i+1$, and go to Step 3.2.

### 5.4 Step 4.

Update *pbest* and *gbest* by evaluating and comparing the fitness value with their previous values.

### 5.5 Step 5.

If the termination criteria are satisfied, then stop. Otherwise, return to Step 2.

## 6. Simulation Results

In this section, the potential of the proposed method is investigated by applying to two different systems. The first system (Case A) is a 10-unit 12-hour system with smooth cost functions [4] and the second system (Case B) is a 10-unit 24-hour system with non-smooth cost functions [2], where the details are given in Appendix. The parameters for experiments are set to: initial inertia weight ($w_{\max}$) = 0.9, final inertia weight ($w_{\min}$) = 0.4, acceleration constants ($c_1$,$c_2$) = 2.05, constriction factor (k) = 0.73, and number of runs = 30, respectively.

### 6.1 Case A: DED problem with smooth cost function

In this experiment, the parameter settings are: population size (Pop) = 10 and maximum iteration = 10000. Table 1 compares the mean cost, the minimum cost, the maximum cost and the standard deviation of the mean costs obtained from the proposed EPSO algorithm with Linear programming (LP). From the results show that the proposed method outperforms in finding the better solution compared with the LP. Furthermore, the best solution obtained from the proposed methods is illustrated in Table 2.

### 6.2 Case B: DED problem with non-smooth cost function

In this case, the simulation parameters of the proposed method are population size (Pop) = 20, and maximum number of generations = 20000, respectively. To compare with other methods, the simulation results of the proposed EPSO algorithm is recorded and tabulated with the results of the Evolutionary Programming (EP) [2], the hybrid method between Evolutionary Programming and Sequential Quadratic Programming (EP-SQP) [2], the modified hybrid EP-SQP (MHEP-SQP) [22], the hybrid method between PSO and SQP (PSO-SQP) [3], the PSO-SQP method with the "crazy"[1] particle (PSO-SQP(C)) [3], and the deterministically guided PSO (DGPSO) [23] in Table 3. From the simulation results show that the proposed EPSO method is more efficient and effective than other algorithms as it requires less number of populations and iterations to obtain solution with high quality. The best solution obtained from the proposed methods is tabulated in Table 4 as well.

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1 *Crazy particle* is re-initialization the velocities of the particle randomly when a random number (0,1) is less than or equal to the predefined probability.

## 7. Conclusion

This paper presents the application of an enhanced PSO (EPSO) to Dynamic Economic Dispatch (DED) problem considering characteristic of smooth and non-smooth cost functions. The EPSO not only utilizes the basic PSO algorithm in order to seek the optimal solution of DED problem, but also enhances its performance by using a modified heuristic method to deal with the constraints and increase the possibility of generating feasible solutions. For investigation and validation of its potential, the EPSO has been implemented and tested on both 10-unit systems (smooth and non-smooth cost functions). Additionally, the experimental results are also compared to other methods. From the simulation results, it can be concluded that the EPSO outperforms others with respect to the quality, the stability as well as reliability of the solutions.

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Table 1 Comparison of calculation results obtained from the proposed EPSO method and LP method for Case A

| Method | Pop | Iteration | Run | Mean cost ($) | Min. cost ($) | Max. cost ($) | Std. Dev |
|---|---|---|---|---|---|---|---|
| LP [4] | - | - | - | - | 2,196,939 | - | - |
| EPSO | 10 | 10000 | 30 | 2,196,534.979 | 2,196,534.946 | 2,196,535.031 | 0.022 |

Table 2 The best simulation result obtained from the proposed EPSO method for Case A

| Hour | Load (MW) | Generation schedule (MW) | | | | | | | | | | Total cost ($) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | U1 | U2 | U3 | U4 | U5 | U6 | U7 | U8 | U9 | U10 | |
| 1 | 5560 | 238.565 | 354.856 | 411.599 | 475.440 | 415.681 | 546.819 | 620 | 643 | 907.880 | 946.160 | 173395.234 |
| 2 | 5620 | 243.611 | 360.850 | 417.259 | 481.753 | 421.280 | 555.778 | 620 | 643 | 919.161 | 957.307 | 176057.858 |
| 3 | 5800 | 261.674 | 380.850 | 439.172 | 505.441 | 442.310 | 588.583 | 620 | 643 | 920 | 998.970 | 184199.278 |
| 4 | 5810 | 262.516 | 382.780 | 440.345 | 506.633 | 443.286 | 590.330 | 620 | 643 | 920 | 1001.111 | 184658.722 |
| 5 | 5990 | 280.960 | 402.779 | 462.277 | 530.308 | 464.422 | 623.122 | 620 | 643 | 920 | 1043.131 | 193057.392 |
| 6 | 6041 | 286.766 | 409.900 | 469.090 | 537.802 | 471.119 | 633.322 | 620 | 643 | 920 | 1050.000 | 195481.820 |
| 7 | 6001 | 282.036 | 404.562 | 463.653 | 531.743 | 465.695 | 624.957 | 620 | 643 | 920 | 1045.354 | 193578.540 |
| 8 | 5790 | 260.640 | 379.988 | 437.935 | 503.932 | 441.160 | 586.838 | 620 | 643 | 920 | 996.507 | 183740.580 |
| 9 | 5680 | 249.510 | 367.501 | 424.638 | 489.595 | 428.181 | 566.645 | 620 | 643 | 920 | 970.929 | 178744.593 |
| 10 | 5540 | 237.049 | 353.167 | 409.386 | 473.454 | 413.669 | 543.943 | 620 | 643 | 904.165 | 942.167 | 172512.592 |
| 11 | 5690 | 250.508 | 368.693 | 425.789 | 490.788 | 429.586 | 568.327 | 620 | 643 | 920 | 973.309 | 179195.013 |
| 12 | 5750 | 256.633 | 375.519 | 433.104 | 498.729 | 436.407 | 579.165 | 620 | 643 | 920 | 987.443 | 181913.324 |
| Total | | | | | | | | | | | | 2196534.946 |

Table 3 Comparison of calculation results obtained from the proposed EPSO method and various methods for Case B

| Method | Pop | Iteration | Run | Mean cost ($) | Min. cost ($) | Max. cost ($) | Std. Dev |
|---|---|---|---|---|---|---|---|
| EP [2] | 80 | 50000 | 20 | 1,048,638 | - | - | - |
| EP-SQP [2] | 60 | 30000 | 20 | 1,035,748 | 1,031,746 | - | - |
| MHEP-SQP [22] | 60 | 30000 | 30 | 1,031,179 | 1,028,924 | - | - |
| PSO-SQP [3] | 100 | 30000 | 30 | 1,031,371 | 1,030,773 | 1,053,983 | - |
| PSO-SQP(C) [3] | 100 | 30000 | 30 | 1,028,546 | 1,027,334 | 1,033,983 | - |
| DGPSO [23] | 60 | 30000 | 30 | 1,030,183 | 1,028,835 | - | - |
| EPSO | 20 | 20000 | 30 | 1,027,890.72 | 1,023,772.46 | 1,031,088.35 | 1773.96 |

Table 4 The best simulation result obtained from the proposed EPSO method for Case B

| Hour | Load (MW) | Generation schedule (MW) | | | | | | | | | | Total cost ($) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | U1 | U2 | U3 | U4 | U5 | U6 | U7 | U8 | U9 | U10 | |
| 1 | 1036 | 150.002 | 135.000 | 309.122 | 60.018 | 73.000 | 57.266 | 129.591 | 47.001 | 20.001 | 55 | 28426.765 |
| 2 | 1110 | 226.629 | 135.002 | 286.381 | 109.954 | 73.001 | 57.016 | 100.004 | 47.001 | 20.011 | 55 | 30601.874 |
| 3 | 1258 | 303.137 | 214.998 | 206.411 | 60.000 | 121.882 | 99.978 | 129.594 | 47.000 | 20.000 | 55 | 33658.338 |
| 4 | 1406 | 379.895 | 222.277 | 286.410 | 61.143 | 74.737 | 129.937 | 129.597 | 47.000 | 20.004 | 55 | 36340.309 |
| 5 | 1480 | 453.087 | 222.253 | 274.241 | 60.000 | 73.000 | 145.765 | 129.588 | 47.065 | 20.000 | 55 | 38196.635 |
| 6 | 1628 | 456.529 | 231.803 | 314.741 | 60.296 | 122.938 | 160.000 | 129.616 | 77.065 | 20.013 | 55 | 41730.711 |
| 7 | 1702 | 456.809 | 311.802 | 316.050 | 60.001 | 73.002 | 160.000 | 129.596 | 89.728 | 50.012 | 55 | 43319.614 |
| 8 | 1776 | 456.496 | 391.801 | 299.930 | 70.443 | 73.002 | 160.000 | 129.590 | 119.728 | 20.012 | 55 | 44836.610 |
| 9 | 1924 | 457.566 | 460.000 | 340.000 | 120.433 | 122.923 | 128.121 | 129.913 | 90.001 | 20.043 | 55 | 48498.206 |
| 10 | 2072 | 456.496 | 460.000 | 335.248 | 170.432 | 172.733 | 122.459 | 129.589 | 120.000 | 50.043 | 55 | 52131.528 |
| 11 | 2146 | 456.501 | 459.999 | 339.573 | 220.189 | 222.599 | 122.508 | 129.591 | 119.997 | 20.043 | 55 | 53766.530 |
| 12 | 2220 | 456.498 | 459.999 | 325.022 | 241.245 | 222.604 | 159.999 | 129.591 | 119.999 | 50.043 | 55 | 55511.821 |
| 13 | 2072 | 380.330 | 460.000 | 300.801 | 192.414 | 222.649 | 160.000 | 129.602 | 120.000 | 51.205 | 55 | 52117.379 |
| 14 | 1924 | 456.495 | 396.794 | 282.968 | 170.402 | 172.729 | 122.453 | 129.590 | 116.364 | 21.206 | 55 | 48248.056 |
| 15 | 1776 | 379.874 | 396.794 | 292.788 | 120.413 | 172.729 | 122.448 | 129.590 | 86.364 | 20.000 | 55 | 44399.908 |
| 16 | 1554 | 303.244 | 316.797 | 306.440 | 120.557 | 122.867 | 123.136 | 129.592 | 56.365 | 20.003 | 55 | 39911.449 |
| 17 | 1480 | 226.623 | 309.530 | 286.665 | 110.460 | 172.729 | 122.401 | 129.591 | 47.001 | 20.000 | 55 | 38018.815 |
| 18 | 1628 | 303.255 | 309.910 | 318.944 | 61.634 | 222.633 | 160.000 | 129.623 | 47.000 | 20.001 | 55 | 41269.818 |
| 19 | 1776 | 379.869 | 389.910 | 339.951 | 66.708 | 222.597 | 123.432 | 129.592 | 48.941 | 20.002 | 55 | 44703.204 |
| 20 | 2072 | 457.193 | 460.000 | 340.000 | 116.708 | 224.560 | 160.000 | 129.600 | 78.941 | 49.999 | 55 | 52126.215 |
| 21 | 1924 | 456.494 | 391.025 | 319.916 | 120.416 | 222.625 | 159.993 | 129.590 | 48.942 | 20.001 | 55 | 48004.760 |
| 22 | 1628 | 379.868 | 311.025 | 297.333 | 70.416 | 172.734 | 145.033 | 129.591 | 47.000 | 20.000 | 55 | 41287.766 |
| 23 | 1332 | 303.244 | 231.025 | 240.835 | 60.002 | 122.858 | 122.447 | 129.589 | 47.000 | 20.000 | 55 | 35029.865 |
| 24 | 1184 | 226.628 | 151.026 | 299.131 | 60.004 | 73.000 | 122.619 | 129.591 | 47.000 | 20.000 | 55 | 31636.281 |
| Total | | | | | | | | | | | | 1023772.456 |

# Appendix

Table A.1 Units data for test Case A (10-unit 12-hour system) [4]

| Gen. | $P_{min}$ (MW) | $P_{max}$ (MW) | a | b | c | UR | DR |
|---|---|---|---|---|---|---|---|
| 1 | 155 | 360 | 0.03720 | 26.4408 | 180 | 20 | 25 |
| 2 | 320 | 680 | 0.03256 | 21.0771 | 275 | 20 | 25 |
| 3 | 323 | 718 | 0.03102 | 18.6626 | 352 | 50 | 50 |
| 4 | 275 | 680 | 0.02875 | 16.8894 | 792 | 50 | 50 |
| 5 | 230 | 600 | 0.03223 | 17.3998 | 440 | 50 | 50 |
| 6 | 350 | 748 | 0.02064 | 21.6180 | 348 | 50 | 50 |
| 7 | 220 | 620 | 0.02268 | 15.1716 | 588 | 100 | 100 |
| 8 | 225 | 643 | 0.01776 | 14.5632 | 984 | 100 | 150 |
| 9 | 350 | 920 | 0.01644 | 14.3448 | 1260 | 100 | 150 |
| 10 | 450 | 1050 | 0.01620 | 13.5420 | 1260 | 100 | 150 |

Table A.2 Units data considering valve-point loading for test Case B (10-unit 24-hour system) [2]

| Gen. | $P_{min}$ (MW) | $P_{max}$ (MW) | a | b | c | e | f | UR | DR |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 150 | 470 | 0.00043 | 21.60 | 958.20 | 450 | 0.041 | 80 | 80 |
| 2 | 135 | 460 | 0.00063 | 21.05 | 1313.6 | 600 | 0.036 | 80 | 80 |
| 3 | 73 | 340 | 0.00039 | 20.81 | 604.97 | 320 | 0.028 | 80 | 80 |
| 4 | 60 | 300 | 0.00070 | 23.90 | 471.60 | 260 | 0.052 | 50 | 50 |
| 5 | 73 | 243 | 0.00079 | 21.62 | 480.29 | 280 | 0.063 | 50 | 50 |
| 6 | 57 | 160 | 0.00056 | 17.87 | 601.75 | 310 | 0.048 | 50 | 50 |
| 7 | 20 | 130 | 0.00211 | 16.51 | 502.70 | 300 | 0.086 | 30 | 30 |
| 8 | 47 | 120 | 0.00480 | 23.23 | 639.40 | 340 | 0.082 | 30 | 30 |
| 9 | 20 | 80 | 0.10908 | 19.58 | 455.60 | 270 | 0.098 | 30 | 30 |
| 10 | 55 | 55 | 0.00951 | 22.54 | 692.40 | 380 | 0.094 | 30 | 30 |

Table A.3 Load demand for test Case A [4]

| Hour | Load (MW) | Hour | Load (MW) |
|---|---|---|---|
| 1 | 5560 | 7 | 6001 |
| 2 | 5620 | 8 | 5790 |
| 3 | 5800 | 9 | 5680 |
| 4 | 5810 | 10 | 5540 |
| 5 | 5990 | 11 | 5690 |
| 6 | 6041 | 12 | 5750 |

Table A.4 Load demand for test Case B [2]

| Hour | Load (MW) | Hour | Load (MW) | Hour | Load (MW) | Hour | Load (MW) |
|---|---|---|---|---|---|---|---|
| 1 | 1036 | 7 | 1702 | 13 | 2072 | 19 | 1776 |
| 2 | 1110 | 8 | 1776 | 14 | 1924 | 20 | 2072 |
| 3 | 1258 | 9 | 1924 | 15 | 1776 | 21 | 1924 |
| 4 | 1406 | 10 | 2072 | 16 | 1554 | 22 | 1628 |
| 5 | 1480 | 11 | 2146 | 17 | 1480 | 23 | 1332 |
| 6 | 1628 | 12 | 2220 | 18 | 1628 | 24 | 1184 |